Physics 242: Homework Problem Set 2
Due: February 16, 2011

1 The “Nesting Function” and Variation of Phase Space with Dimension.

The nesting function is defined as

\[ \xi(\vec{Q}) = \frac{1}{N} \sum_{\vec{k}} \delta(\varepsilon_k - \varepsilon_F) \delta(\varepsilon_{k+Q} - \varepsilon_F) = C \int_{\mathcal{L}} \frac{d\mathcal{L}_k}{|\vec{v}_k \times \vec{v}_{k+Q}|}, \]  

(1)

where \( N \) is the (usual) number of unit cells in the normalization volume, \( C \) is a constant you should include properly, and \( \vec{v}_k \) is the electron velocity, the gradient of \( \varepsilon_k \). \( \mathcal{L} \) is the line of intersection of the two surfaces specified by the \( \delta \)-functions.

Factoid: note that averaging \( \xi(Q) \) over \( Q \) gives \( N(\varepsilon_F)^2 \), which is the total number of transitions from \( \vec{k} \) on the Fermi surface to state \( \vec{k} + \vec{Q} \) on the Fermi surface. Thus \( \xi(Q) \) just describes the distribution of those processes. Typically peaks occurs where there is Fermi surface nesting (the cross product is small), and is enhanced further if the velocities are lower than the mean.

For a free electron dispersion \( \varepsilon_k = \frac{\vec{k}^2}{2m} \), evaluate \( \xi(Q) \) for both two dimension and three dimensions, to observe how “phase space” enters differently in the two cases and affects the result. You’ll always need 3-vectors for the cross product, so for 2D just assume there is no \( k_z \) dependence of \( \varepsilon_k \). Plot the results; a hand drawn sketch is sufficient.

Hint: sketching the intersection Fermi surfaces and doing some elementary geometry makes this a straightforward problem.

Finally: think about what happens in one dimension; describe briefly.