1. $T = \frac{\hbar^2 c}{eH} \frac{\partial}{\partial \varepsilon} A(\varepsilon, k_z)$.

for free electron $A = \pi k^2 = \pi \frac{2mE}{\hbar^2}$

$$\Rightarrow T = \frac{\hbar^2 c}{eH} \frac{\partial}{\partial \varepsilon}\left(\pi \frac{2mE}{\hbar^2}\right) = \boxed{\frac{2\pi mc}{eH}} \quad \#$$

2.

(a) $\varepsilon(\vec{k}) = c + \frac{\hbar^2}{2}(\vec{k}-\vec{k}_0)\cdot M^{-1}(\vec{k}-\vec{k}_0)$. (d

Choose the coordinate that $M_{xy}^{-1} = M_{yx}^{-1} = 0$.

$$\Rightarrow M^{-1} = \begin{pmatrix} M_{xx}^{-1} & 0 & M_{zx}^{-1} \\ 0 & M_{yy}^{-1} & M_{zy}^{-1} \\ M_{xz}^{-1} & M_{yz}^{-1} & M_{zz}^{-1} \end{pmatrix}, \text{ define } \vec{k}-\vec{k}_0 = \vec{k}'$$

$$\Rightarrow \varepsilon(\vec{k}) = c + \frac{\hbar^2}{2}\left[k_x'^2 M_{xx}^{-1} + k_y'^2 M_{yy}^{-1} + k_z'^2 M_{zz}^{-1} + 2k_x' k_z' M_{xz}^{-1} + 2k_y' k_z' M_{yz}^{-1}\right]$$

$$\Rightarrow \quad = c + \frac{\hbar^2}{2}\left[\left(M_{xx}^{-\frac{1}{2}} k_x' + \frac{M_{xz}^{-1}}{M_{xx}^{-\frac{1}{2}}} k_z'\right)^2 - \frac{M_{xz}^{-1\,2} k_z'^2}{M_{xx}^{-1}} + \left(M_{yy}^{-\frac{1}{2}} k_y' + \frac{M_{yz}^{-1} k_z'}{\sqrt{M_{yy}^{-1}}}\right)^2 - \frac{M_{yz}^{-2} k_z'^2}{M_{yy}^{-1}} + k_z'^2 M_{zz}^{-1}\right]$$

$$\Rightarrow \frac{2}{\hbar^2}(\varepsilon - c) + k_z'^2\left(\frac{M_{xz}^{-1\,2}}{M_{xx}^{-1}} + \frac{M_{yz}^{-1\,2}}{M_{yy}^{-1}} - M_{zz}^{-1}\right) = \left[\left(\sqrt{M_{xx}^{-1}}\, k_x' + \frac{M_{xz}^{-1}}{\sqrt{M_{xx}^{-1}}} k_z'\right)^2 + \left(\sqrt{M_{yy}^{-1}}\, k_y' + \frac{M_{yz}^{-1} k_z'}{\sqrt{M_{yy}^{-1}}}\right)^2\right]$$

For A is a ellipse area

$$1 = \frac{k_x'^2}{a^2} + \frac{k_y'^2}{b^2} \Rightarrow A = \pi ab$$

in our case $A = \frac{\pi}{\sqrt{M_{xx}^{-1} M_{yy}^{-1}}}\left(\frac{2}{\hbar^2}(\varepsilon - c) + k_z'^2\left(\frac{M_{xz}^{-1\,2}}{M_{xx}^{-1}} + \frac{M_{yz}^{2}}{M_{yy}^{-1}} - M_{zz}^{-1}\right)\right)$.

$$\Rightarrow m^* = \frac{\hbar^2}{2\pi}\frac{\partial A}{\partial \varepsilon} = \frac{\hbar^2}{2\pi}\frac{\pi}{\sqrt{M_{xx}^{-1} M_{yy}^{-1}}} \cdot \frac{2}{\hbar^2} = (M_{xx}^{-1} M_{yy}^{-1})^{-\frac{1}{2}} \Rightarrow \boxed{m^* = \sqrt{\frac{\det M}{M_{zz}}}} \ \#$$

(b)

Choose the coordinate which make M is diagonlize.

$$\Rightarrow \quad \varepsilon(\vec{k}) = C + \frac{\hbar^2}{2}\left(\frac{k_x^2}{m_x} + \frac{k_y^2}{m_y} + \frac{k_z^2}{m_z}\right).$$

$$C_v = \frac{\pi^2}{3} k_B^2 T g(\varepsilon_F) \quad \text{for free electron} \quad g(\varepsilon_F) = \frac{m}{\pi^2\hbar^2} \frac{(2m E_F)^{\frac{1}{2}}}{\hbar} \quad - ①$$

For the electron near the band minimum.

$$g(\varepsilon_F) = 2 \int \frac{d^3k}{(2\pi)^3} \delta(\varepsilon - \varepsilon(\vec{k})). \quad \text{let } q_i = \frac{k_i}{\sqrt{m_i}} \quad d^3q = \frac{d^3k}{\sqrt{m_x m_y m_z}}$$

$$= \frac{(m_x m_y m_z)^{\frac{1}{2}}}{4\pi^2} \cdot \int d^3q \, \delta(\varepsilon - \frac{\hbar^2}{2}\vec{q}^{\,2})$$

$$= \frac{(\det M)^{\frac{1}{2}}}{\pi^2} \int q^2 \, dq \, \frac{\delta(q - q(\varepsilon))}{|\nabla \varepsilon|_{q(\varepsilon)}} = \boxed{\frac{(\det M)^{\frac{1}{2}}}{\pi^2\hbar^2} \cdot \frac{(2E_F)^{\frac{1}{2}}}{\hbar}} - ②$$

Compare ① & ② get $\boxed{m^* = (\det M)^{1/3}}$

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(a)

$$\varepsilon(\vec{k}) = \varepsilon_0 + \frac{\hbar^2}{2}(\vec{k}-\vec{k}_0)\,M^{-1}(\vec{k}-\vec{k}_0).$$

$$\Rightarrow \quad \vec{v} = \frac{1}{\hbar}\frac{\partial \varepsilon(\vec{k})}{\partial \vec{k}} = \frac{\hbar}{2}\left(M^{-1}(\vec{k}-\vec{k}_0) + (\vec{k}-\vec{k}_0)\cdot M^{-1}\right)$$

for $M^{-1}_{ij} = \partial^2 \varepsilon / \partial k_i \partial k_j \Rightarrow M^{-1}_{ij} = M^{-1}_{ji}$

$$\Rightarrow \quad \vec{v} = \hbar M^{-1}(\vec{k}-\vec{k}_0).$$

time derivate of above equation $\Rightarrow \dot{\vec{v}}(k) = \hbar M^{-1}\dot{\vec{k}}$

also for $\hbar\dot{\vec{k}} = -e\vec{E} \Rightarrow \dot{\vec{v}}(k) = -M^{-1}e\vec{E} - \boxed{\frac{\vec{v}}{\tau}}$ ← include collision.

For stable state $\dot{\vec{v}} = 0 \Rightarrow \vec{v} = -\tau M^{-1}e\vec{E}$

$$\Rightarrow J = -env = \tau e^2 M^{-1}\vec{E}$$

$$\Rightarrow \sigma = \boxed{n\tau e^2 M^{-1}} \quad ※$$

(b).

Choose the coordinate which diagonalize mass tensor

$$\Rightarrow \quad M\cdot\frac{d\vec{v}}{dt} = -e\left(\vec{E} + \frac{\vec{v}}{c}\times\vec{H}\right), \quad \vec{H} = H\hat{z}$$

$$\Rightarrow \begin{pmatrix} M_{xx} & 0 & 0 \\ 0 & M_{yy} & 0 \\ 0 & 0 & M_{zz} \end{pmatrix}\begin{pmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{pmatrix} = \begin{pmatrix} -eE_x - \frac{e}{c}v_yH \\ -eE_y + \frac{e}{c}v_xH \\ -eE_z \end{pmatrix}$$

$$M_{xx}\dot{v}_x = -eE_x - \frac{e}{c}v_yH$$

$$M_{yy}\dot{v}_y = -eE_y + \frac{e}{c}v_xH$$

$$M_{zz}\dot{v}_z = -eE_z$$

$$\Rightarrow \quad M_{xx}\ddot{\vartheta}_x = -\frac{e}{c}\dot{V}_y H = -\frac{eH}{cM_{yy}}\left(-eE_y + \frac{e}{c}V_x H\right)$$

$$\Rightarrow \quad \ddot{\vartheta}_x = \frac{e^2 E_y H}{cM_{xx}M_{yy}} - \frac{e^2 H^2}{c^2 M_{xx}M_{yy}} V_x \Rightarrow V_x(t) = A\sin(wt) + B\cos(wt)$$

$$\text{with } W = \frac{eH}{c}(M_{xx}M_{yy})^{-\frac{1}{2}}$$

The same for $\ddot{V}_y$

$$\ddot{V}_y = \frac{-e^2 E_x H}{cM_{xx}M_{yy}} - \frac{e^2 H^2}{c^2 M_{yy}M_{xx}} V_y \Rightarrow V_y(t) = C\sin(wt) + D\cos(wt)$$

$$\text{with } W = \frac{eH}{c}(M_{xx}M_{yy})^{-\frac{1}{2}}$$

$$\Rightarrow \boxed{m^* = (M_{xx}M_{yy}) = \sqrt{\frac{\det M}{M_{zz}}}}$$