P. 1 (a)

At Fermi surface

$$\varepsilon_F = \varepsilon^0_{K/2} - |U_K| + \Delta \doteq \varepsilon^0_{K/2} + \frac{\hbar^2 k^2}{2m} \pm \left(4\varepsilon^0_{K/2} \cdot \frac{\hbar^2 k^2}{2m} + |U_K|^2\right)^{1/2}$$

$$\Rightarrow \quad \Delta - |U_K| = \frac{\hbar^2 k^2}{2m} \pm \left(4\varepsilon^0_{K/2} \frac{\hbar^2 k_{\parallel}^2}{2m} + |U_K|^2\right)^{1/2}$$

if $0 < \Delta < 2|U_K|$, 1) Fermi surface in lower band

2) k in Bragg plane $\Rightarrow \vec{k}_{\parallel} = 0$

$$\Rightarrow \quad \Delta - |U_K| = \frac{\hbar^2 k^2}{2m} - \sqrt{|U_K|^2} \Rightarrow k^2 = \frac{2m\Delta}{\hbar^2} \Rightarrow \rho = \sqrt{\frac{2m\Delta}{\hbar^2}}$$ #

Density of state

$$g(\varepsilon) = 2\sum_n \int \frac{d^D k}{(2\pi)^D} \delta(\varepsilon - \varepsilon_n(\vec{k}))$$

for free electron, $\varepsilon_n(\vec{k}) = \frac{\hbar^2 k^2}{2m}$ & $\delta\left(\varepsilon - \frac{\hbar^2 k^2}{2m}\right) = \frac{\delta\left(k - \sqrt{\frac{2m\varepsilon}{\hbar^2}}\right)}{\sqrt{\frac{2\varepsilon\hbar^2}{m}}}$

For 1D: $g(\varepsilon) = 2\int \frac{dk}{2\pi} \delta\left(k - \sqrt{\frac{2m\varepsilon}{\hbar^2}}\right) \Big/ \sqrt{\frac{2\varepsilon\hbar^2}{m}} = \frac{1}{\pi\hbar}\left(\frac{m}{2\varepsilon}\right)^{1/2}$ #

For 3D $\quad g(\varepsilon) = 2\int \frac{4\pi k^2 dk}{(2\pi)^3} \frac{\delta\left(k - \left(\frac{2m\varepsilon}{\hbar^2}\right)^{1/2}\right)}{\sqrt{2m\hbar^2/m}}$

$$= \frac{m}{\pi^2 \hbar^3}(2m\varepsilon)^{1/2}$$ #