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HW#5 Phys 240a
Condensed Matter
October 21, 2015

8.1 a)

$$\psi_l = \begin{cases} e^{iKx} + re^{-iKx}, & x \le -\frac{a}{2} \\ te^{iKx}, & x \ge +\frac{a}{2} \end{cases}$$

$$\psi_r = \begin{cases} te^{-iKx}, & x \le -\frac{a}{2} \\ e^{-iKx} + re^{iKx}, & x \ge +\frac{a}{2} \end{cases}$$

$\psi \approx \psi$

$$\psi(x) = A\psi_l(x) + B\psi_r(x)$$

$$\psi(x+a) = e^{ika}\psi(x), \quad x = -\frac{a}{2} \Rightarrow \psi(+\tfrac{a}{2}) = e^{ika}\psi(-\tfrac{a}{2})$$

$$\Rightarrow Ate^{iK\frac{a}{2}} + B\left(e^{-iK\frac{a}{2}} + re^{iK\frac{a}{2}}\right) = e^{ika}\left[A\left(e^{-iK\frac{a}{2}} + re^{iK\frac{a}{2}}\right) + Bte^{iK\frac{a}{2}}\right]$$

$$\hookrightarrow A\left[te^{iK\frac{a}{2}} - e^{ia(k-\frac{1}{2}K)} - re^{ia(k+\frac{1}{2}K)}\right] = B\left[te^{ia(k+\frac{1}{2}K)} - e^{-iK\frac{a}{2}} - re^{iK\frac{a}{2}}\right] \quad (1)$$

$$\psi'(\tfrac{a}{2}) = e^{ika}\psi'(-\tfrac{a}{2})$$

$$\Rightarrow Ate^{iK\frac{a}{2}} + B\left(-e^{-iK\frac{a}{2}} + re^{iK\frac{a}{2}}\right) = e^{ika}\left[A\left(e^{-iK\frac{a}{2}} - re^{iK\frac{a}{2}}\right) - Bte^{iK\frac{a}{2}}\right]$$

$$\hookrightarrow A\left[te^{iK\frac{a}{2}} - e^{ia(k-\frac{1}{2}K)} + re^{ia(k+\frac{1}{2}K)}\right] = B\left[-te^{ia(k+\frac{1}{2}K)} + e^{-iK\frac{a}{2}} - re^{iK\frac{a}{2}}\right] \quad (2)$$

① + ② :

$$A\left[2te^{iK\frac{a}{2}} - 2e^{ia(k-\frac{1}{2}K)}\right] = B\left[-2re^{iK\frac{a}{2}}\right]$$

$$\Rightarrow B = A\,\frac{e^{ia(k-\frac{1}{2}K)} - te^{iK\frac{a}{2}}}{re^{iK\frac{a}{2}}}$$

① - ② :

$$A\left[-2re^{ia(k+\frac{1}{2}K)}\right] = B\left[2te^{ia(k+\frac{1}{2}K)} - 2e^{-iK\frac{a}{2}}\right]$$

$$\Rightarrow A\left[re^{ia(k+\frac{1}{2}K)}\right] = \frac{A}{re^{iK\frac{a}{2}}}\left(e^{-iK\frac{a}{2}} - te^{ia(k+\frac{1}{2}K)}\right)\left(e^{ia(k-\frac{1}{2}K)} - te^{iK\frac{a}{2}}\right)$$

$$\hookrightarrow r^2e^{ia(k+K)} = e^{ia(k-K)} - t - te^{2iak} + t^2e^{ia(k+K)}$$

$$e^{iKa}(r^2-t^2)e^{ika} + e^{-iKa}(-e^{ika}) = -t(1+e^{2ika})$$

$$e^{ika}\left[e^{iKa}(r^2-t^2) - e^{-iKa}\right] = e^{ika}\left[-t(e^{-ika}+e^{ika})\right]$$

$$e^{iKa}(r^2-t^2) - e^{-iKa} = -t(2\cos ka)$$

$$\Rightarrow \boxed{\cos ka = \frac{t^2-r^2}{2t}e^{iKa} + \frac{1}{2t}e^{-iKa}}$$

free electron : $t=1,\ r=0,\ K=k$

$$\Rightarrow \cos ka = \frac{1}{2}(e^{ika}+e^{-ika}) = \cos ka \quad \checkmark$$