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HW#12 Phys 204A
Condensed Matter
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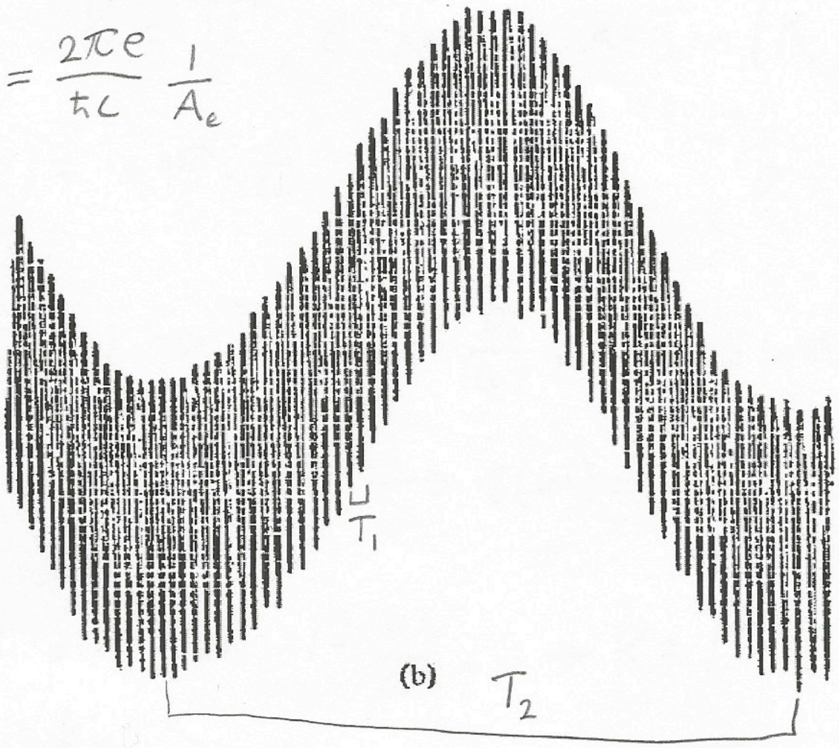
14.2

$$T \equiv \Delta\left(\frac{1}{H}\right) = \frac{2\pi e}{\hbar c}\,\frac{1}{A_e}$$



$$\frac{T_2}{T_1} \approx 50 \Rightarrow \boxed{\frac{A_{max}}{A_{min}} \approx 50}$$

15.2 $r = \frac{|E^r|^2}{|E^i|^2} = \frac{|1-K|}{|1+K|} = \frac{(1-n)^2+k^2}{(1+n)^2+k^2}$

with $K = \sqrt{\varepsilon}$, $n = Re(K)$, $k = Im(K)$

$\varepsilon(\omega) = 1 + 4\pi i \frac{\sigma}{\omega}$, $\sigma(\omega) = \frac{\sigma_0}{1-i\omega\tau}$, $\sigma_0 = \frac{ne^2\tau}{m}$

$\omega\tau >> 1 \Rightarrow \sigma(\omega) \approx -\frac{\sigma_0}{i\omega\tau} \Rightarrow \varepsilon(\omega) \approx 1 + 4\pi i\left(-\frac{\sigma_0}{i\omega^2\tau}\right)$

define $\omega_p^2 = \frac{4\pi ne^2}{m} \longrightarrow \varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$

1) $\omega < \omega_p$: Then $1 - \frac{\omega_p^2}{\omega^2} < 0 \Rightarrow K = \sqrt{\varepsilon}$ is pure imaginary!

$n = 0$, $k = \frac{K}{i}$

$\Rightarrow r = \frac{(1-0)^2+k^2}{(1+0)^2+k^2} = \boxed{1}$ ✓

2) $\omega > \omega_p$: Then $1 - \frac{\omega_p^2}{\omega^2} > 0 \Rightarrow K = \sqrt{\varepsilon}$ is pure real!

$n = K$, $k = 0$

$\Rightarrow r = \frac{(1-n)^2}{(1+n)^2} \longrightarrow n = \sqrt{1-\delta} \approx 1 - \frac{1}{2}\delta$ for $\delta = \left(\frac{\omega_p}{\omega}\right)^2 << 1$

$\hookrightarrow r \approx \frac{[1-(1-\frac{1}{2}\delta)]^2}{[1+(1-\frac{1}{2}\delta)]^2} \approx \frac{[\frac{1}{2}\delta]^2}{[2]^2} = \left(\frac{\delta}{4}\right)^2 = \boxed{\left(\frac{\omega_p^2}{4\omega^2}\right)^2}$ ✓

15.3

For electron.

$E_e = (p^2c^2 + m_e^2c^4)^{\frac{1}{2}}$, $p_e = p$

For photon.

$E = \hbar\omega$, $p_p = \hbar k$.

energy conservation.

$$\Rightarrow (p_i^2c^2 + m_e^2c^4)^{\frac{1}{2}} + \hbar\omega = (p_f^2c^2 + m_e^2c^4)^{\frac{1}{2}} \quad \text{—①}$$

Momentum conservation. $p_f = p_i + \hbar k$. —②

Put ② into ① and divid each side by c, we get.

$$(p_i^2 + m_e^2c^2)^{\frac{1}{2}} + \hbar \frac{\omega}{c}^{\nearrow k} = ((p_i + \hbar k)^2 + m_e^2c^2)^{\frac{1}{2}} \quad \text{(square both sides).}$$

$$\Rightarrow \cancel{p_i^2} + \cancel{m_e^2c^2} + \cancel{\hbar^2k^2} + 2\hbar k(p_i^2 + m_e^2c^2)^{\frac{1}{2}} = \cancel{p_i^2} + 2\hbar k p_i + \cancel{\hbar^2k^2} + \cancel{m_e^2c^2}.$$

$$\Rightarrow 2\hbar k(p_i^2 + m_e^2c^2)^{\frac{1}{2}} = 2\hbar k p_i \Rightarrow p_i = (p_i^2 + m_e^2c^2)^{\frac{1}{2}}$$

$\Rightarrow m_e^2c^2 = 0$, which is not possible, hence the equation of energy conservation of photon adsorption is not valid