$$J_\alpha = -\frac{e}{\Omega}\sum_{\vec{k}} v_\alpha(\vec{k}) F(\vec{k}) = \sigma_{\alpha\beta} E_\beta + \nu_{\alpha\beta} \nabla_\beta T + \cdots$$

$$\varepsilon\left(\vec{k} - \frac{\tau}{\hbar}\vec{F}_{ext}\right) \tilde{=} \varepsilon(\vec{k}) - \frac{\tau}{\hbar}\vec{F}_{ext}\cdot\vec{\nabla}_k \varepsilon(\vec{k}) = \varepsilon(\vec{k}) - \tau\vec{F}_{ext}\cdot\vec{v}(\vec{k})$$

$$\Rightarrow J_\alpha = -\frac{e}{\Omega}\sum_{\vec{k}} v_\alpha(\vec{k}) \cdot f\left[T - \vec{v}(\vec{k})\tau\cdot\vec{\nabla}T ; \varepsilon\left(\vec{k} - \frac{\tau}{\hbar}\vec{F}_{ext}\right)\right]$$

$$\tilde{=} -\frac{e}{\Omega}\sum_{\vec{k}} v_\alpha(\vec{k}) \left[f(T, \varepsilon(\vec{k})) - \underbrace{\tau\,\vec{v}(\vec{k})\cdot\vec{\nabla}T \frac{\partial f(T,\varepsilon)}{\partial T}}_{①} - \underbrace{\tau\vec{F}_{ext}\cdot\vec{v}(\vec{k})\frac{\partial f(T,\varepsilon)}{\partial \varepsilon}}_{②} \right]$$

* For $\sigma_{\alpha\beta}$, we look at ②

$$\Rightarrow \frac{-e}{\Omega}\sum_{\vec{k}} v_\alpha(\vec{k})\cdot\vec{F}_{ext}\cdot\vec{v}(\vec{k})\frac{(-\partial f)}{\partial \varepsilon}, \quad \text{let } \vec{F}_{ext} = -e\vec{E}$$

$$= \underline{\frac{-e^2\tau}{\Omega}\sum_{\vec{k}}\left(v_\alpha(\vec{k})\, v_\beta(\vec{k})\frac{\partial f}{\partial \varepsilon}\right) E_\beta} = \sigma_{\alpha\beta} E_\beta$$

* For $\nu_{\alpha\beta}$, we look at ①,

$$\frac{\partial f(T,\varepsilon)}{\partial T} = -\frac{e^{\frac{\varepsilon-u}{kT}}\left(-\frac{\varepsilon-u}{kT^2}\right)}{\left[1+\exp\left(\frac{\varepsilon-u}{k_B T}\right)\right]^2}, \quad \frac{\partial f}{\partial \varepsilon} = \frac{e^{\frac{\varepsilon-u}{kT}}\left(\frac{1}{kT}\right)}{\left[1+\exp\left(\frac{\varepsilon-u}{k_B T}\right)\right]^2}, \Rightarrow \frac{\partial f}{\partial T} = \frac{-(\varepsilon-u)}{T}\frac{\partial f}{\partial \varepsilon}$$

$$① = -\frac{e}{\Omega}\sum_{\vec{k}} v_\alpha(\vec{k})\left[-\tau\, v(\vec{k})\cdot\vec{\nabla}T\cdot\frac{\partial f(T,\varepsilon)}{\partial T}\right]$$

$$= -\frac{e}{\Omega}\sum_{\vec{k}} v_\alpha(\vec{k})\left[-\tau\, v(\vec{k})\cdot\vec{\nabla}T\cdot\left(\frac{-(\varepsilon-u)}{T}\cdot\frac{\partial f}{\partial \varepsilon}\right)\right.$$

$$= \underline{-\frac{e\tau}{\Omega T}\sum_{\vec{k}}\left[v_\alpha(\vec{k})\, v_\beta(\vec{k})(\varepsilon-u)\frac{\partial f}{\partial \varepsilon}\right]\cdot\nabla_\beta T} = \nu_{\alpha\beta}\cdot\nabla_\beta T \;\; \#\#$$