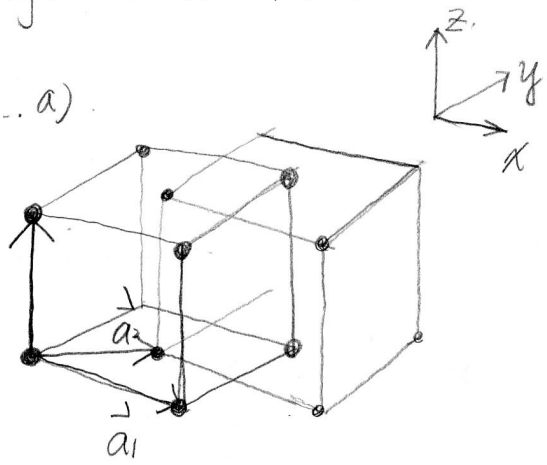
hys 140A HW1

a)



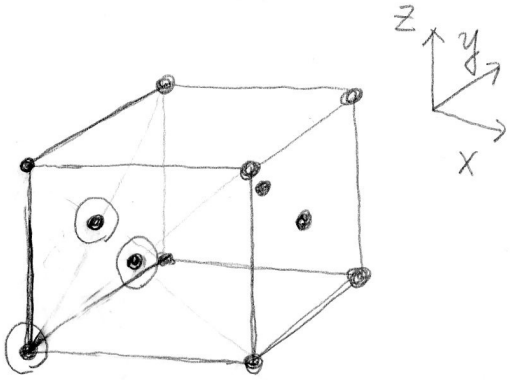
This is a Bravias lattice with primitive vector

$\vec{a}_1 = a\hat{x}$
$\vec{a}_2 = \frac{a}{2}(\hat{x}+\hat{y})$
$\vec{a}_3 = a\hat{z}$

or

$\vec{a}_1 = \frac{a}{2}(\hat{x}-\hat{y})$
$\vec{a}_2 = \frac{a}{2}(\hat{x}+\hat{y})$
$\vec{a}_3 = a\hat{z}$

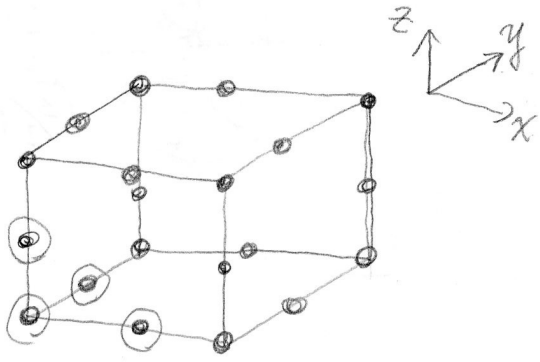
b)



Side center cubic is not a Bra lattice, but can be represented b simple cubic with three points ba

$(0,0,0)$, $(\frac{a}{2},0,\frac{a}{2})$, $(0,\frac{a}{2},\frac{a}{2})$.

c)

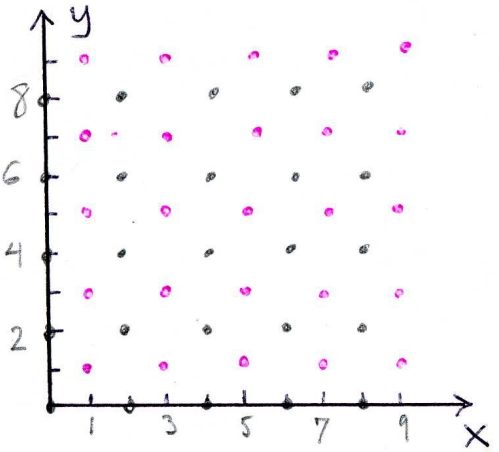


edge center cubic is not a Bra lattice, but can be represented simple cubic with four points

$(0,0,0)$, $(0,0,\frac{a}{2})$, $(0,\frac{a}{2},0)$, $(\frac{a}{2},0,0)$

: 4.2

a)



$\vec{R} = n_1\hat{x} + n_2\hat{y} + n_3\hat{z}$

n_1, n_2, n_3 even

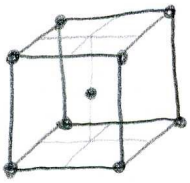
– or –

n_1, n_2, n_3 odd

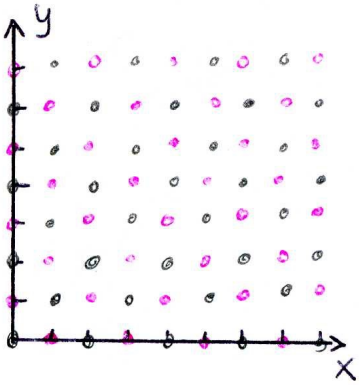
grey: $n_3 = 0, 2, 4, \ldots$

pink: $n_3 = 1, 3, 5, \ldots$

This forms the Body-center cubic:



b)



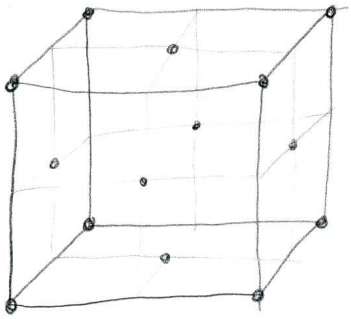
$\vec{R} = n_1\hat{x} + n_2\hat{y} + n_3\hat{z}$

$n_1 + n_2 + n_3$ even

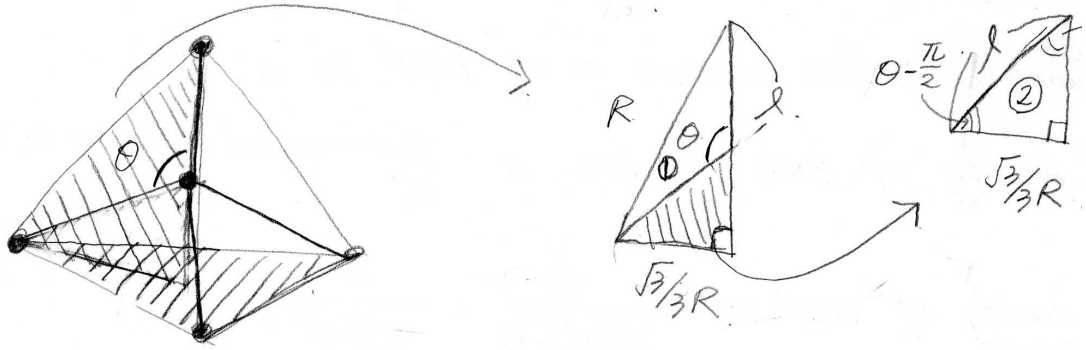
grey: n_3 even

pink: n_3 odd

This forms the face-centered cubic:



3



From triangle ①, we get 1) $R^2 = \ell^2 + \ell^2 - 2\ell^2\cos\theta$ (law of cosin)

$\Rightarrow R^2 = 2\ell^2(1-\cos\theta)$ — eq(1).

From triangle ②, we get 2) $\frac{\ell}{\sin\frac{\pi}{2}} = \frac{\sqrt{3}/3 R}{\sin(\pi-\theta)}$ (law of Sin)

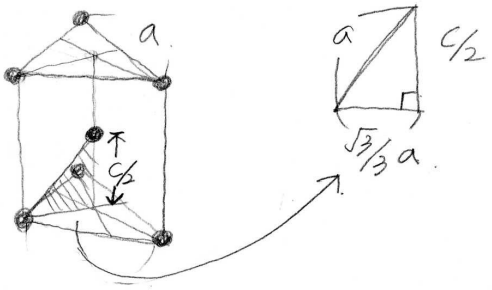
$\Rightarrow R = \sqrt{3}\ell\sin\theta$ → eq(2).

put eq2 into eq1 $\Rightarrow 3\ell^2\sin^2\theta = 2\ell^2(1-\cos\theta)$.

$3\ell^2(1-\cos\theta)(1+\cos\theta) = 2\ell^2(1-\cos\theta)$

$\Rightarrow 3+3\cos\theta = 2 \Rightarrow 3\cos\theta = -1 \Rightarrow \theta =$ Cos-1(1/3)

4 (a)



$\Rightarrow a^2 = (\frac{c}{2})^2 + (\frac{\sqrt{3}}{3}a)^2$

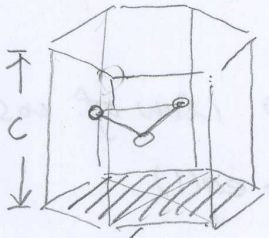
$\Rightarrow \frac{c^2}{4} = \frac{2}{3}a^2 \Rightarrow c = \sqrt{\frac{8}{3}}a$

Q 4(b)

Assume the lattice constant a of HCP is a'.

⇒ The density of cubic structure is $\frac{2}{a^3}$ ← number of atoms in BCC cubi

The density of HCP is $\frac{6}{3\sqrt{3}(a')^2 \cdot c} = \frac{6}{6\sqrt{2}a'^3}$.



bottom area $= 3\sqrt{3}(a')^2$.

$\Rightarrow \frac{6}{6\sqrt{2}(a')^3} = \frac{2}{a^3} \Rightarrow a' = \left(\frac{a^3}{\sqrt{2}}\right)^{\frac{1}{3}} = .3.72(\text{Å})$.