1. Electronic Response. 60 points.

The “Lindhard susceptibility” $\chi_o(Q, \omega)$ describes the response of a system of ‘non-interacting’ electrons. It is the density-density response function in $(Q, \omega)$ space [Fourier transformed from $(\vec{r}, \vec{r}'; t - t')$ space] given by

$$\chi_o(Q, \omega) = \frac{1}{N} \sum_k \frac{f_k - f_{k+Q}}{\varepsilon_{k+Q} - \varepsilon_k - \hbar\omega - i\eta}.$$ 

Here $\varepsilon_k = \hbar^2 k^2 / 2m$, $\eta$ is a positive infinitesimal that provides the imaginary part, and $N$ is the number of unit cells in the normalization box (i.e. number of $\vec{k}$ points). The zero-temperature Fermi-Dirac function $f_k \equiv f(\varepsilon_k)$ is unity if $\varepsilon_k < \varepsilon_F$, and zero if $\varepsilon_k < \varepsilon_F$.

(i) Calculate $\chi_o(Q, \omega)$ for a three-dimensional gas of electrons. You should consider using the fact that

$$f_k - f_{k+Q} = f_k(1 - f_{k+Q}) - (1 - f_k)f_{k+Q}. \quad (1)$$

Keep all constants $\hbar$ and $m$ in the final answer. It is conventional to scale wavevectors by $k_F$ and energies by $\varepsilon_F$ to simplify notation by working with dimensionless quantities; insert these constant back into the result. Show work; you can find the result many places.

(ii) Plot $\chi_o(Q, \omega)$, both real and imaginary parts to get an idea of its behavior. This can be done most elegantly by a surface plot in the $(Q, \omega)$ plane, but can also be done by plotting versus $\omega$ for a few (4-6) values of $Q$, positioned on the same plot. Using the scaled variables is good for presentation.

(iii) Calculate $\chi_o(Q, \omega)$ for a one-dimensional system.