1. \( B = 2.4 \times 10^4 \text{ erg/cm}^2 \)
   \( a = 5.65 \text{Å} \)
   Spring constant: \( C \)

(i) \[ C_{eq} = \frac{C}{2} \]

For 2 springs in series, we get an effective spring constant of:

\[ C_{eq} = \left( \frac{1}{C} + \frac{1}{C} \right)^{-1} = \frac{C}{2}. \]

We need to get the total number of series of a unit cell:

Within unit cell: 3 full series
On the surfaces: 2 series, 6 surfaces
On the edges: 1 \( \frac{1}{2} \) each, 12 edges

So, \( N_{total} = 3 + 6 \times \frac{2}{2} + 12 \times \frac{1}{4} = 12 \)

So, the total energy of the cube, when stretched/compressed + length \( \Delta l \), is:

\[ E = 12 \times \frac{C}{2} \times \left( \frac{a}{2} (a-x) \right) \]

\[ = 3C (a - \frac{1}{4})^2 \]

The bulk modulus is:

\[ B = \frac{V \Delta E}{\Delta V} \]

So,

\[ B = V \frac{2}{3V} \left[ - \frac{C}{2} (aV^{1/3}) \right] \]

\[ = \frac{2}{3} \left[ - 2C \left( aV^{-2/3} - V^{-1/3} \right) \right] \]

\[ = 2C \left( \frac{1}{3} V^{-1/3} + \frac{a}{3} V^{-2/3} \right) \]

\[ = \frac{2C}{3} \left( 2aV^{-1/3} - V^{-1/3} \right) \]

Near equilibrium \( (\Delta \approx 0) \),

\[ B = \frac{2C}{3} \left( \frac{a}{2} - \frac{1}{a} \right) = \frac{2C}{3a} \]

So,

\[ C = \frac{3aB}{2} = 1.5 (5.65 \times 10^{-8} \text{ cm}) (2.4 \times 10^4 \text{ erg/cm}^2) \]

\[ \approx 2.83 \times 10^4 \text{ erg/cm}^2 \]
(ii) Optical phonon mode for 1D system in a basis:

\[ \omega_{\text{ph}}(k) = \sqrt{\frac{2\alpha (M_1 + M_2)}{M_1 M_2}} \]

\[ = \sqrt{\frac{2 (2.03 \times 10^4 \text{ erg/cm}^2)(95.45 + 22.99)}{95.45 \times 22.99 / (6.023 \times 10^{23})}} \]

\[ = 4.17 \times 10^5 \text{ s}^{-1} \]

Use also for speed of sound:

\[ \omega_{\text{ac}}(k) = \sqrt{\frac{\alpha}{\alpha + (M_1 + M_2)}} \]

\[ C_s = \sqrt{\frac{\alpha}{\alpha + (M_1 + M_2)}} \]

\[ = 5.8 \times 10^3 \text{ m/s} \]

3. In a quasi-2D system, the spring constant in the \( z \) direction of the plan is small, so the phonon energies in that direction are low. In the planar directions, the 2D long waves are dominant at high \( \omega \). The behavior look as such:

\[ D(\omega) \]

\[ = \text{Debye component } + \text{ Einstein component} \]

\[ \text{The specific heat curves for } \text{ both } \text{ the 2 models look like this (not } f) : \]

\[ C_v \]

\[ \theta_E \sim \omega_{\text{ac}} \]

\[ \text{overall} \]

\[ \text{So, the } C_v \text{ curve should go like } T^3, \text{ with an Einstein saturation at } T = \theta_E : \]
2. (i) Two-dimensional analogy of Eq. 13.62 (Marden p. 320):

\[ D(\omega) = \int \frac{2\pi kdk}{\pi^2} \delta(\omega - ck) \]

\[ = \frac{1}{2\pi} \int \frac{ck d(ck)}{c^2} \delta(\omega - ck) \]

\[ = \frac{\omega}{2\pi c^2} \]

So in the Debye model,

\[ D(\omega) = \frac{\omega}{2\pi c^2} \Theta(\omega_p - \omega) \]

N = total # of modes.

Then

\[ 2N = A \int_{0}^{\omega_p} D(\omega) d\omega \]

\[ = \frac{1}{2} \frac{\omega_p}{\pi c^2} A \]

\[ = \frac{k_p^2}{4\pi c} A \]

\[ \therefore k_p = \sqrt{\frac{8\pi N}{A}} = \sqrt{\frac{8\pi N}{A}} \]

(ii) In 1D,

\[ D(\omega) = \int \frac{d(ck)}{2\pi c} \delta(\omega - ck) \]

\[ = \frac{\omega}{2\pi c} \]

In 2D,

\[ D(\omega) = \frac{\omega}{2\pi c} \]

In 3D,

\[ D(\omega) = \frac{3\omega}{2\pi c^2} \left[ \text{Eq. (13.67)} \right] \]

(iii) \( C_a = \int_0^{\omega_p} d\omega D(\omega) \frac{\omega}{\beta T} e^{\beta \omega} - 1 \)

\[ = A \int_0^{\omega_p} \frac{d(\beta k\omega)}{\beta k} \frac{A_k^2 \omega^2}{2\pi c^2 \beta k} \frac{e^{\beta \omega} - 1}{e^{\beta \omega} - 1} \frac{\omega}{\beta T} e^{\beta \omega} \]

\[ = A \int_0^{\omega_p} \frac{dx x e^x}{(e^{\beta x} - 1)^2} \frac{k^2 T^2}{2\pi c^2 k^2} \]

\[ \therefore C_a = \frac{A k^2 T^2}{2\pi c^2 k^2} \int_0^{\omega_p} \frac{x^2 e^x}{(e^{\beta x} - 1)^2} dx \]

\[ C_a \sim T^2, \text{ where as } C_v \sim T^3 \text{ in 3D.} \]