Physics 240B: Homework Problem Set 1
Due: January 14, 2008

1. Number Operator Algebra. 10 points.
For a fermionic number annihilation operator a and its adjoint, the creation operator $a^\dagger$, the number operator is $\hat{n} = a^\dagger a$. Demonstrate that $\hat{n}^2 = \hat{n}$. [Note: two operators are equivalent if each of their matrix elements are identical.]

2. Second Quantization of a Two State System. 40 points.
Consider a system of fermions for which there are two single-particle states, call them A and B.
(i) **Obtain the matrix representations** of the creation/annihilation operators a, b, analogous to what was done for the one-state system in class.
(ii) find the representations of the product operators $a^\dagger a, b^\dagger b, ab^\dagger$. Provide an interpretation of the first two of these.
Recall that, for the one-state problem,

$$a = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad a^\dagger = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \qquad N = a^\dagger a = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}. \tag{1}$$

Note that, for two particles, there are four "configurations" (states in 2nd-quantizationland), corresponding to $n_a = 0$ or 1, $n_b = 0$ or 1. Your operators a, b should do exactly, and only, what you need them to do.

Hint: to begin, specify your states specifically and symbolically, call them
$\phi_1 \leftrightarrow n_a = 0, n_b = 0,$
$\phi_2 \leftrightarrow n_a = 1, n_b = 0,$
$\phi_3 \leftrightarrow n_a = 0, n_b = 1,$
$\phi_4 \leftrightarrow n_a = 1, n_b = 1.$
Then use what you know the the matrix elements need to be to construct the representations.