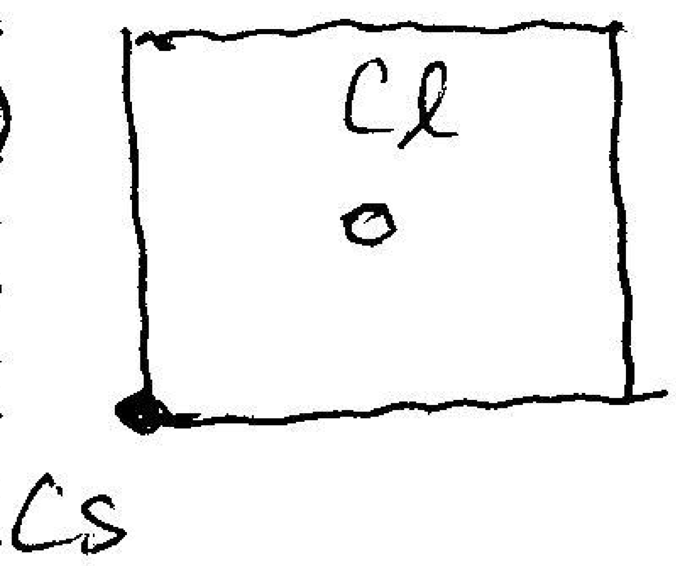
2. $$\underset{\sim}{G}_1 = 2\pi \frac{\underset{\sim}{R}_2 \times \underset{\sim}{R}_3}{\underset{\sim}{R}_1 \cdot \underset{\sim}{R}_2 \times \underset{\sim}{R}_3} \quad \text{cyclic} \Rightarrow \boxed{\underset{\sim}{G}_i = 2\pi \frac{\underset{\sim}{R}_j \times \underset{\sim}{R}_k}{\underset{\sim}{R}_i \cdot \underset{\sim}{R}_j \times \underset{\sim}{R}_k}}$$

3. (a) $$S(\underset{\sim}{k}) = \sum_j^{\text{cell}} f_j \, e^{i \underset{\sim}{k} \cdot \underset{\sim}{\tau}_j}$$

- sum over all atom j in unit cell
- f_j is the atomic form factor (scattering strength)
- $\underset{\sim}{\tau}_j$ is position within the cell.

(sometimes written with normalization factor. Either is ok.)

(b) 

f_{Cs}, f_{Cl} are different

$$S(k) = f_{Cs} + f_{Cl}\, e^{i(k_x a/2 + k_y a/2)}$$

5. (a) $$\langle H \rangle = \int H(\varepsilon) f(\varepsilon)\, d\varepsilon \quad \text{where}$$

- $H(\varepsilon)$ is any function of ε that is <u>smooth</u>
- $f(\varepsilon) = \left[e^{-\beta(\varepsilon - \mu)} + 1 \right]^{-1}$ = Fermi-Dirac distribution fn.

(b) $C_V = \frac{\pi^2}{3} k_B^2 T D(\varepsilon_F)$. linear in T

$D(\varepsilon_F)$ is DOS evaluated at ε_F.

(c) The classical gas has an energy of $\frac{1}{2} k_B T$ for each degree of freedom. So M particles has

$$\varepsilon^{cl} = \frac{3}{2} M k_B T \text{ of energy}, \quad C_V^{cl} = \frac{3}{2} k_B M / V \quad (\text{per unit volume}).$$

Since on average $D(\varepsilon) \sim 1/W \leftarrow$ bandwidth $\approx$ a few eV,

$$C_V^{qm} \simeq \frac{\pi^2}{3} k_B \left(\frac{k_B T}{W} \right), \text{ and for a degenerate gas } \frac{k_B T}{W} \ll 1.$$

This reduction is due to Pauli exclusion: only particles in states within $\sim k_B T$ of E_F can be thermally excited.