PHY 240A HW5, prob. 2

2. (i)

$$\varepsilon_k = 2t_a \cos k_x a + 2t_b \cos k_y b + 2t_c \cos k_z c \,.$$

(ii) Since no term contains (say) both k_x & k_y, the (inverse) mass tensor is diagonal. Then

$$\frac{1}{m_\alpha} = \frac{1}{\hbar^2} \frac{\partial^2 \varepsilon_k}{\partial k_\alpha^2}\bigg|_{\underset{\sim}{k}=0} = -\frac{2t_\alpha a_\alpha^2}{\hbar^2} \cos k_\alpha a_\alpha \bigg|_{\underset{\sim}{k}=0} \qquad a_\alpha = (a, b, c).$$

$$= -\frac{2t_\alpha a_\alpha^2}{\hbar^2} \,.$$

So effective mass is $\propto \frac{1}{t_\alpha a_\alpha^2}$. Narrower bands (smaller t) have higher mass.

Also, note the "strange" minus sign. If $t_\alpha > 0$, then the effective mass is negative. Turns out this is OK, in fact it is right (next quarter).