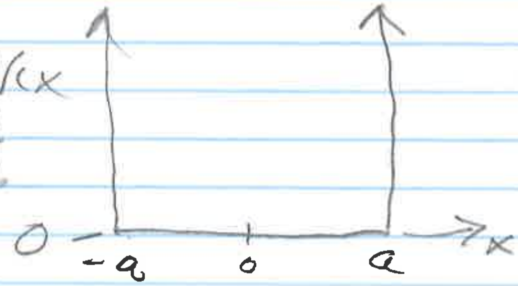
Phy 215C Midterm # 2 May 25, 2017

(2) Dirac Electron in an Infinite Square Well.

In flat zero potential, we know the form of solutions. Just use spin up, spin "symmetry" is not disturbed.



$$\psi = e^{ikx - iEt/\hbar} \begin{bmatrix} 1 \\ 0 \\ \eta_\pm \\ 0 \end{bmatrix} \begin{matrix} \}\chi \\ \\ \}\Phi \end{matrix} \qquad \eta_\pm = \frac{\gamma_k}{1 \pm \sqrt{1+\gamma_k^2}}$$

$\boxed{\text{-k state has same energy.}}$

with normalization factor $N_k^{-1/2}$ if normaliz'n is required.

As in the non-relativistic square well, $\psi \to 0$ at a & $-a$.

Wvfn: $A e^{ikx} + B e^{-ikx}$ [other factors are same throughout

Ground state will be symmetric: $\psi = A \cos kx \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$ ← from above

At $x = a$ (or $x = -a$): $\cos ka = 0 \Rightarrow ka = \frac{\pi}{2}[2q+1]$ for integer q.

Ground state: $q = 0 \Rightarrow k_0 a = \frac{\pi}{2}$, $k_0 = \frac{\pi}{2a}$, $\frac{\hbar^2 k_0^2}{2m} = \left(\frac{\pi}{2}\right)^2 \frac{\hbar^2}{2ma^2}$

Energy: $\boxed{E_0 = \pm\sqrt{m^2c^4 + \hbar^2 k_0^2 c^2} = \pm mc^2\left[1 + \left(\frac{k_0}{\varkappa_c}\right)^2\right]^{1/2}}$ $\varkappa_c = \frac{mc}{\hbar}$ = Compton wvvector

This is extra.

At what value of a is $|E_0| = 2mc^2$? [should be a "3" here (in sq. root)]

At $\hbar^2 k_0^2 c^2 = m^2c^4$, $k_0^2 = \frac{m^2c^2}{\hbar^2} = \varkappa_c^2 \Rightarrow \frac{\pi}{2a} = \frac{mc}{\hbar}$, $a = \frac{\pi\hbar}{2mc} \sim \frac{3}{2}\frac{1}{\varkappa_c} \sim \frac{3}{2}\frac{a_B}{137}$.

or $a \simeq 10^{-2} a_B \approx 0.005 \text{Å}$. ← should be divided by $\sqrt{3}$.

(ii) Ground state kinetic energy is 10% of mc^2: $\frac{\hbar^2 k_0^2}{2m} \simeq \frac{1}{10} mc^2 \simeq \frac{\hbar^2}{2m}\frac{\pi^2}{4a^2}$ ← good enough approx.

$a^2 = \frac{\hbar^2\pi^2}{2m}\frac{10}{mc^2} = 5\pi^2 \frac{1}{\varkappa_c^2} \approx 5 \times 10 \times \lambda_c^2$, $\lambda_c = \alpha^2 a_0$, Compton wvlength

$$\boxed{a \simeq 7\alpha a_0 = \frac{7}{137} \times 0.5 \text{Å} \simeq \frac{1}{20} \times 0.5 \text{Å} = 0.025 \text{Å}}$$