PHY 215C, Quantum Mechanics: Homework Set #7

Due: June 1, 2017

May 21, 2017

1. Angular Momentum in Dirac Theory. 20 points.
Given the Dirac Hamiltonian $H_D = c\alpha \cdot \vec{p} + \beta mc^2 + V(r)$ with a spherical potential:
(i) Show that $[\vec{L}, H_D]$ is non-zero, so the orbital angular momentum is not a constant of the motion.
(ii) show that $[\vec{S}, H_D]$ also is not zero, but is such that the total angular momentum $\vec{J} = \vec{L} + \vec{S}$ does commute, and therefore is a constant of the motion. The spin operator is given by

$$\vec{S} = \frac{\hbar}{2} \begin{bmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{bmatrix}.$$

2. Relativistic Electron in a Harmonic Potential. 15 points.
Consider a Dirac electron confined to a 3D harmonic oscillator potential $V(\vec{r}) = (m\omega^2/2)\vec{r}^2$. You should realize that the derivation of relativistic corrections in Shankar Sec. 20.2 applies to general spherical potential wells, not just the H atom, so you don't need to do that again. Using Eq. 20.2.28:
(i) evaluate the relativistic correction to the kinetic energy (p^4 term) in the ground state, expressed as a fraction of the zero point energy. Use (of course) the non-relativistic eigenstates.
(ii) work out the explicit form of the spin-orbit term for this harmonic oscillator (4th term in Eq. 20.2.28).
(iii) Argue, "semiclassically" if you want, what will be the lowest state with a non-zero expectation value of the spin-orbit interaction.

3. Electromagnetics in Dirac Theory. 5points.

Prove two operator identities that are used in Sec. 20.2 (and elsewhere in physics).
(a) For vector operators $\vec{C}, \vec{D}$ that commute with the Pauli matrices $\vec{\sigma}$ (such as $\vec{r}$, $\vec{p}$, $\vec{L}$, etc.), show that

$$\sigma \cdot \vec{C} \sigma \cdot \vec{D} = \vec{C} \cdot \vec{D} + i\sigma \cdot \vec{C} \times \vec{D}.$$

(b) Show that

$$\Pi \times \Pi = \frac{iq\hbar}{c}\vec{B}, \qquad \text{where} \qquad \Pi \equiv \vec{p} - \frac{q\vec{A}}{c}.$$

Here $\vec{B}, \vec{A}$ are the magnetic field and vector potential, respectively.
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4. Scattering from a Spherical Shell. 20 points.
Consider a particle of mass μ scattering from a δ-function spherical shell potential $V(r) = aV_o\delta(r - a)$.

(i) Calculate the scattering amplitude in the Born approximation. For $ka = 10\pi$ (a large value of this dimensioness parameter), sketch the angluar (θ) dependence. Do the same for $ka = \pi/2$. [One can also observe surprising behavior at fixed angle versus ka.]

(ii) Now, perform a partial wave analysis to find the $\ell = 0$ phase shift δ_o for $ka << 1$, where this phase shift should dominate. We should discuss in class why this result does not seem to agree with the result from part (ii) [perhaps there is some reason it should not].