# Homework #6, PHY 215C

Due: May18, 2017

**Problem 1. Scattering from a square well potential. 20 points.**

Consider of particle of mass $\mu$ and wavevector $\vec{k}$ impinging on a "square well" (actually, spherical) of constant depth $V_o$ and range $r_o$, which in the previous assignment you treated in the Born approximation.

(i) Following the formalism of Eq. (19.5.22) and below, find the expression for the phase shifts $\delta_\ell(k)$. Keep in mind that the spherical Neumann function $n_\ell(x)$ blows up at the origin, impacting your choice of wavefunction inside the potential region.

(ii) Consider the limiting regime of near-vanishing potential, $V_o \to 0$. Since only $\delta_{\ell=0}$ should be appreciable in this regime (at least for energy not too small), check whether taking this limit indeed gives the correct result for the cross section. The issue is that the matching you did in part (i) seems to become iffy. (Why?)

**Problem 2. Scattering from a square well potential, yet again. 10 points.**

Given this same square well potential, the differential cross section is given in Ex. 19.3.2. That is the result to be used. [It is interesting that in Born approximation the sign of $V_o$ is irrelevant, but that is not part of this problem.]

(i) Consider that $V_o$ gets large as $r_o$ goes to zero in a way that the "integrated potential" $(4\pi/3)r_o^2 V_o$ stays constant. How does the differential cross section, and total cross section, behave in this limit? Is it physical, and possibly correct, or clearly unphysical? Discuss what may be the cause of the behavior you find.

**Problem 3. Continuity equation in Dirac theory. 10 points.**

Derive the continuity equation for Dirac theory given in Exercise 20.1.1.

**Problem 4. Dirac electron in a uniform magnetic field. 20 points.**

(i) Shankar Exercise (20.2.2): solve for the *exact* energy levels of a Dirac electron in a uniform magnetic field, using the given vector potential. Hint: see Exercise 12.3.8, which is the same problem for the Schrödinger electron.

(ii) Take the $B \to 0$ limit to obtain the energies of a free Dirac electron.