Homework #3, PHY 215C

Due: April 25, 2017

Problem 1. Interaction Representation. 10 points.

You are given a system described by the Hamiltonian $H^0 + V(t)$. The time-dependent Schrödinger equation gives the differential equation that the Schrödinger state $|\psi(t)>_S$ must satisfy, that is very old news.

(i) First, transform to the Interaction Representation and obtain the differential equation that the IR state $|\psi>_I$ must satisfy.

(ii) Then, given the eigenstate expansion

$$|\psi(t)>_I = \sum_n c_n(t)|n>$$

in terms of the time-independent eigenstates $|n>$ of H^0, obtain the differential equation that the coefficients $c_n(t)$ must satisfy.

Problem 2. A periodically driven two-level system. 20 points.

A simple system with only two states is governed by the Hamiltonian

$$H^0 + V(t); \quad H^0 = \bar{E}\sigma_o + \Delta\sigma_z; \quad V(t) = V_o[cos(\omega t)\sigma_x + sin(\omega t)\sigma_y],$$

in terms of the Pauli matrices and the identity σ_o. $\bar{E} = (E_1 + E_2)/2; \Delta = E_1 - E_2)/2$. Note: this is NOT a perturbation problem.

(i) First, explicitly write the Hamiltonian as a 2×2 matrix so you can see it in that form also, which may even be helpful.

(ii) Using whatever representation you wish (and describe which you are using), derive the equations of motion (differential equations) for the expansion coefficients of the state of the system.

(iii) Given that at time t=0 the system is entirely in state $|1>$ of H^0 which has states 1 and 2, solve for the state of the system at a later time t.

(iv) Describe the main features of the solution, or items of interest.

Problem 3. β-decay of Tritium. 30 points.

A tritium atom (a proton and two neutrons in the nucleus, plus the neutralizing electron) in the ground state undergoes β-decay into $^3He^+$, a positively charged He atom. One of the neutrons has decayed into a proton+electron, and the new electron has very rapidly left the vicinity.

(i) Begin by writing down the "perturbation" that will describe this instantaneous process.

(ii) Use the sudden approximation to obtain the probability that the initial electron will be in the $2s$ state of He immediately after the decay.

(iii) Now consider the longer time behavior. In this process the "perturbation" (the change in potential from the nucleus) is turned on at time zero and remains. Use 1st order P.T. to calculate the probability that the $1s$ electron of H will at time $t > 0$ be in the $2s$ state of He.

(iv) From the result of part (iii), let $t \to 0$ in your expression. Does the result agree with that from the sudden approximation? Discuss why or why not.