PHY 210: Homework Problem A
to be discussed in class

1 Numerical study of the convergence of a Taylor Series.

The Taylor series! for $sin\ x$, which converges for all values of x, is given by

$$\begin{aligned} sin\ x &= \sum_{i=1}^{\infty} (-)^{(i-1)} \frac{x^{2i-1}}{(2i-1)!} \qquad (1) \\ &= x - \frac{x^3}{1 \times 2 \times 3} + \frac{x^5}{1 \times 2 \times 3 \times 4 \times 5} - \frac{x^7}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7} + ... \end{aligned}$$

Investigate how this series converges for a few values of x. Specifically, calculate and plot the sequence of partial sums $S_j(x_o)$ vs. j, given by

$$S_j(x_o) = \sum_{i=1}^{J} (-)^{(i-1)} \frac{x_o^{2i-1}}{(2i-1)!} \qquad (2)$$

for the values $x_o = 1.;\ 1. + 2\pi;\ 1. + 4\pi;\ 1. + 6\pi$. Plot them on a line graph in such a way that they may be compared easily.

2 Rearranging a mathematical expression

Re-code your partial sums in the form

$$S_j(x_o) = x(1 - \frac{x^2}{2 \times 3}(1 - \frac{x^2}{4 \times 5}(1 - \frac{x^2}{6 \times 7}(1 - ......)))) \qquad (3)$$

where the number of terms that are kept correspond to the value of the upper limit J in Eqn. (2). Using single precision arithmetic, explore for what values of $x_\circ$ and/or J a difference can be seen. Which is more accurate? Which is most efficient?