

# Computational Physics 210: Homework #2

## 1 Integrating a Differential Equation

We have gone through a 1st approximation method, and a 2nd approximation method, for integrating a first-order differential equation of the form

$$\frac{dY}{dx} = f(x, Y) \quad (1)$$

where  $Y$  is the (possibly vector) function to be solved for, and  $f(x, Y)$  is a given function. Compare the two approximations for the following two choices for scalar functions  $f$ :

(i)  $f(x, Y) = 2\pi \cos(2\pi x)$ .

(ii)  $f(x, Y) = \frac{\sin(2\pi x)}{2\pi x}$ .

Integrate from  $x=0$  to some distance of your choice, to allow you both compare the two approximations, and to assess when a solution is acceptable. Note that in these two cases there is the simplification  $f(x, Y) \rightarrow (f(x))$ .

For uniformity on this problem, let  $h = 1/N$  denote the step size, corresponding on the number of points  $N$  chosen within each unit interval of  $x$ . You should try integrating well beyond  $x=1$ . The  $2\pi$  factors should make some aspects of the problem more straightforward, viz. the period of  $\cos(2\pi x)$  is unity.

In each case, choose a sensible initial condition (you know the solution in the first case). Almost the same code should work for both, except case (ii) has the issue of how to take care of the  $x \rightarrow 0$  region.

The objective of this problem is to let you explore the solutions and organize results in a useful, pedagogical manner. Plotting quantities will be necessary.