Computational Physics 210: Homework #3

1 Integrating a Differential Equation: Forced Harmonic Oscillator

The equation for a forced harmonic oscillator is

$$\frac{d^2x}{dt^2} + \omega^2 x(t) = g(t), \tag{1}$$

where ω is the natural frequency of the free oscillator, and g is a given forcing function. Use the 2nd order approximation we discussed in class to integrate this equation, first converting it into a system of two coupled first-order differential equations.

Without loss of generality, take $\omega = 2\pi$ (this sets a scale on the time t axis, and makes the period of the free oscillator equal to unity. Make sure you specify π as accurately as possible in your code, that that is not a source of loss of precision. Use initial conditions $x(0) = 0, x'(0) = 1$.

(i) First, do the free oscillator: $g = 0$. Compare with the exact result.

(ii) Second, do the forced oscillator at resonance: $g(t) = \epsilon cos(\omega t)$, with the same ω as above (the resonant frequency) and $\epsilon = 0.1$.

Integrate from t=0 to t=10 (ten periods), plot results, vary the step size until you get converged results.

You will hand in your code, and a written description of what you have observed, supported by plots. It can, of course, be very interesting and educational to change $g(t)$: change the frequency and/or the amplitude, or simply change g to some other function altogether. You should organize your code so such changes can be made immediately (including changes in the initial conditions).