PHYS 140B HW #7

1.

(a) $$\Phi(z) = \frac{1}{\sqrt{2}}\left(e^{ikz} + e^{i(k+q)z}\right)\Psi_0$$

$$\Phi^*(z)\,\Phi(z) = \frac{1}{2}\left(e^{-ikz} + e^{-i(k+q)z}\right)\left(e^{ikz} + e^{i(k+q)z}\right)|\Psi_0|^2$$

$$= \frac{1}{2}\left(1 + e^{iqz} + e^{-iqz} + 1\right)|\Psi_0|^2$$

$$= \frac{1}{2}\left(2 + 2\cos(qz)\right)|\Psi_0|^2$$

$$= \left(1 + \cos(qz)\right)|\Psi_0|^2$$

(b) $$K_\Psi = \int dz\, \Psi^*(z) \cdot \frac{-\hbar^2}{2m}\frac{d^2}{dz^2}\Psi(z)$$

$$= -\frac{\hbar^2}{2m}\cdot(-ik)^2 = \frac{\hbar^2 k^2}{2m}$$

$$K_\Phi = \int dz\, \Phi^*(z) \cdot -\frac{\hbar^2}{2m}\frac{d^2}{dz^2}\Phi(z)$$

$$= \frac{1}{2}\cdot\left(\frac{\hbar^2}{2m}\right)\cdot\left[(k+q)^2 + k^2\right] \sim \frac{\hbar^2 k^2}{2m} + \frac{\hbar^2}{2m}kq$$

$$K_\Phi - K_\Psi = \frac{\hbar^2 kq}{2m}$$

(c) $K_\Phi - K_\Psi = 2\Delta$ and $\xi = 1/q$

$$\Rightarrow 2\Delta = \frac{\hbar^2 k}{2m\xi} = \frac{\hbar^2 k_F}{2m\xi} = \frac{\hbar m v_F}{2m\xi} = \frac{\hbar v_F}{2\xi}$$

$$\therefore \xi = \frac{\hbar v_F}{4\Delta}$$

(d)

	v_F (m/s)	Δ (eV)	$\xi = \frac{\hbar v_F}{4\Delta}$ (m)	
Cs	0.75×10^6			
Ca	1.28×10^6			
Al	2.02×10^6	3.4×10^{-4}	9.78×10^{-7}	9780 Å
Pb	1.82×10^6	27.3×10^{-4}	1.1×10^{-7}	1100 Å

2. $T_c = 125\,K$, $\Theta_D = 250\,K$, $\kappa(T_c) = 50$

(a) energy gap $2\varepsilon(0) = 3.52\, k_B T_c$ eqn. (16.25)

$$= 3.52 \times 8.6173\times10^{-5}\ eV\,K^{-1} \times 125\,K = 3.8\times10^{-2}\ eV$$

(b) upper critical field $H_p(0) = \dfrac{\varepsilon(0)}{\mu_0 \mu_B} = \dfrac{1.9\times10^{-2}\ eV \times 1.6\times10^{-19}\ J/eV}{1.2566\times10^{-6}\ N/A^2 \times 9.274\times10^{-24}\ J/T}$

eqn. (16.33)

$$= 2.6\times10^{8}\ T\cdot A^2/N = 2.6\times10^{8}\ A/m\ (SI)$$

This corresponds to B of 327 T (Gaussian)

(c) coherence length : $H_{c2}(T) = \dfrac{\Phi_0}{2\pi\mu_0 \xi^2(T)}$ eqn. (16.22)

$$\Rightarrow \xi^2(0) = \frac{h/2e}{2\pi\mu_0 H_{c2}(0)} = \frac{6.6261\times10^{-34}\ J\cdot S / 2\times1.6\times10^{-19}\ C}{2\pi\times1.2566\times10^{-6}\ N/A^2 \times 2.6\times10^{8}\ TA^2/N}$$

$$= 10^{-18}\ m^2$$

$$\Rightarrow \xi(0) = 10^{-9}\,m = 10\ \mathring{A}$$

penetration depth : $\kappa(T_c) = 0.96\,\lambda(0)/\xi(0)$

$$\lambda(0) = \frac{\kappa(T_c)\,\xi(0)}{0.96} = \frac{50\times10^{-9}\ m}{0.96} = 5.21\times10^{-8}\ m = 521\ \mathring{A}$$

(d) thermodynamic critical field :

$$H_c(0) = \frac{\Phi_0}{2\pi\mu_0\sqrt{2}\,\lambda(0)\,\xi(T)} = \frac{6.6261\times10^{-34}\ J\cdot S / 2\times1.6\times10^{-19}\ C}{2\pi\times1.2566\times10^{-6}\ N/A^2 \times\sqrt{2}\times5.21\times10^{-17}\ m^2}$$

eqn. (16.18)

$$= 3.56\times10^{6}\ T\cdot A^2/N = 3.56\times10^{6}\ A/m\ (SI)$$ or corresponds to B of 4.5 T

(e) $$\gamma = \frac{\pi^2 k_B}{2T_c} = \frac{\pi^2\times8.6173\times10^{-5}\ eV\,K^{-1}}{2\times125\,K} = 3.4\times10^{-6}\ eV\cdot K^{-2}$$

eqn. (7.26)

$$A = \frac{12\pi^4}{5} k_B \left(\frac{1}{\Theta_D}\right)^3 = \frac{12\pi^4\times8.6173\times10^{-5}\ eV\ K^{-1}}{5\times(250)^3\ K^3} = 1.29\times10^{-9}\ eV\cdot K^{-4}$$

eqn. (5.44)