1.

(a)

$$G_n(0,T) - G_s(0,T) = \frac{\mu_0 H_c(T)^2}{2}$$

@ $T=0$, $H_c(0) = 6.4 \times 10^4$ A/m for Pb

$$\Rightarrow \text{Condensation energy} = \frac{(1.2566\times10^{-6})\times(6.4\times10^4)^2}{2} \ \frac{N}{A^2}\left(\frac{A}{m}\right)^2$$

$$\doteqdot 2573 \ J/m^3$$

$$2573 \ J/m^3 = \frac{2573}{\underset{\uparrow \ J/eV}{1.6\times10^{-19}} \times \underset{\uparrow \ \text{electron}/m^3}{13.2\times10^{28}}} \ \frac{eV}{\text{electron}} \doteqdot 1.22 \times 10^{-7} \ eV/\text{electron}$$

(b)

$$\mathcal{E}(0)\frac{\mathcal{E}(0)}{E_F} = 1.3 \ meV \times \frac{1.3 \ meV}{9.37 \ eV}$$

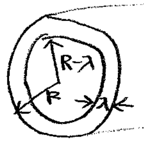
$$\doteqdot 1.8 \times 10^{-7} \ eV$$

Almost the same as in (a).

2.

(a)

$$J_c = \frac{i_c}{A_{eff}}$$



$$A_{eff} = \pi R^2 - \pi (R-\lambda)^2$$

$$= \pi R^2 - \pi R^2 + 2\pi R\lambda - \pi\lambda^2$$

$$= 2\pi R\lambda - \pi\lambda^2 \simeq 2\pi R\lambda$$

$$\Rightarrow J_c = \frac{i_c}{A_{eff}} = \frac{i_c}{2\pi R\lambda} = \frac{H_c}{\lambda}, \quad w/ \ H_c = \frac{i_c}{2\pi R}$$

(Ampere's law)

Hence J_c is independent of R.

(b) For Pb @ $T = 0$, $H_c(0) = 6.4 \times 10^4$ A/m and $\lambda = 39$ nm

$$\therefore \quad J_c(0) = \frac{H_c(0)}{\lambda(0)} = \frac{6.4 \times 10^4}{39 \times 10^{-8}} \frac{A}{m^2} \doteqdot 1.64 \times 10^{12} \ A/m^2$$

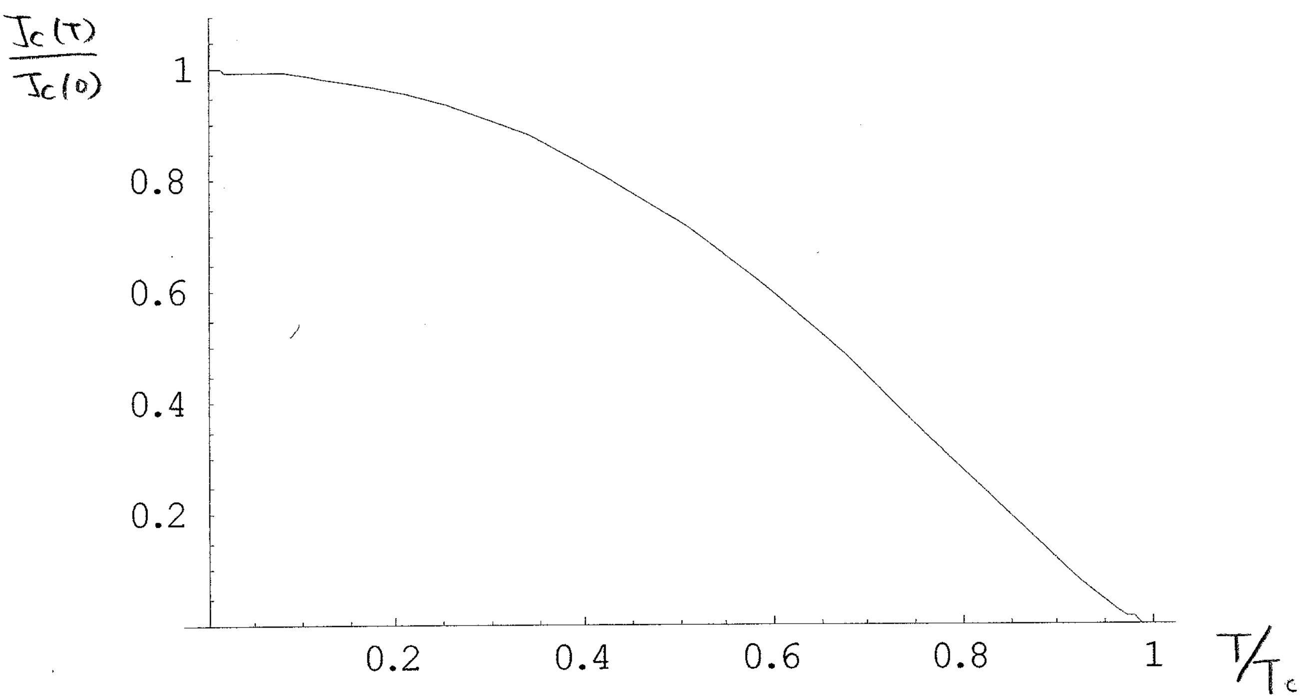
(c)

$$H_c(T) = H_{c0}\left(1 - \frac{T^2}{T_c^2}\right)$$

$$\lambda_L(T) = \frac{\lambda_L(0)}{\sqrt{1 - \left(\frac{T}{T_c}\right)^4}}$$

$$\Rightarrow \quad J_c(T) = \frac{H_c(T)}{\lambda_L(T)} = \frac{H_{c0}\left(1 - \frac{T^2}{T_c^2}\right)\sqrt{1 - \left(\frac{T}{T_c}\right)^4}}{\lambda_L(0)}$$

$$\therefore \quad \frac{J_c(T)}{J_c(0)} = \left[1 - \left(\frac{T}{T_c}\right)^2\right]\sqrt{1 - \left(\frac{T}{T_c}\right)^4}$$



3.

(a) $\Delta F = \alpha A^2 + \frac{1}{2}\beta A^4 + \alpha B^2 + \frac{1}{2}\beta B^4 + \gamma(\hat{A}\hat{B}^* + \hat{A}^*\hat{B})$

$$\hat{A}\hat{B}^* + \hat{A}^*\hat{B} = AB\left(e^{i(\phi_A - \phi_B)} + e^{-i(\phi_A - \phi_B)}\right)$$

$$= 2AB\cos(\phi_A - \phi_B)$$

$$\Rightarrow \frac{\partial \Delta F}{\partial \phi_A} = 0 = -2\gamma AB\sin(\phi_A - \phi_B) \Rightarrow \phi_A = \phi_B + n\pi,\ n = 0, 1$$

then $\cos(\phi_A - \phi_B) \equiv s$, $s = \pm 1$

(b) $\frac{\partial F}{\partial A} = 0 = \alpha A + \beta A^3 + 2\gamma B s$ $\qquad \alpha = a(T - T_c)$

$$\frac{\partial F}{\partial B} = 0 = \alpha B + \beta B^3 + 2\gamma A s$$

One solution : $B = A \Rightarrow \alpha + \beta A^2 + 2\gamma s = 0$, $A^2 = \frac{-\alpha - 2\gamma s}{\beta}$

$$A^2 = \frac{a(T_c - T) - 2\gamma s}{\beta} = a'(T_c - T) - 2\gamma' s$$

$$= (a'T_c - 2\gamma' s) - a'T$$

$$= a'\left(T_c - \frac{2\gamma s}{\beta a'} - T\right)$$

$$= \left(\frac{a}{\beta}\right)(T_c^* - T)$$ renormalized critical temperature T_c^*

(c)

Then $\Delta F = \alpha A^2 + \frac{1}{2}\beta A^4 + 2\gamma A^2 s$

$$= A^2\left\{\alpha + \frac{1}{2}\beta A^2 + 2\gamma s\right\}$$

$$= A^2\left\{-a(T_c - T) + \frac{1}{2}\beta \cdot \frac{a}{\beta}(T_c^* - T) + 2\gamma s\right\}$$

$$= A^2\left\{-a(T_c - T) + \frac{a}{2}(T_c^* - T) + 2\gamma s\right\}$$

lower energy if $2\gamma s < 0$ or $s = -\operatorname{sgn}\gamma$

suppose $\gamma > 0$, then $s = -1$

i.e. $\operatorname{sgn}(s\gamma) = -1 \Rightarrow T_c$ is raised.

EXTRA CREDIT : here is example of useful probing ....

Look for solution w/ $A \neq B$, i.e. $A - B \neq 0$

(recall both A and B are positive)

subtract $\frac{\partial \Delta F}{\partial B} = 0$ from $\frac{\partial \Delta F}{\partial A} = 0$

$$\alpha (A-B) + \beta (A^3 - B^3) - 2s\gamma (A-B) = 0$$

$-2s\gamma = +2\gamma$ from early general result

$$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$$

so divide out $A-B$:

$$\alpha + \beta(A^2 + AB + B^2) = -2\gamma$$

$$\Rightarrow \quad A^2 + AB + B^2 = \frac{1}{\beta}(|\alpha| - 2\gamma)$$

positive

negative until T is sufficiently below T_c that $|\alpha| > 2\gamma$

$$= \frac{1}{\beta}(a(T_c - T) - 2\gamma)$$

$$= \frac{aT_c - 2\gamma}{\beta} - \frac{aT}{\beta}$$

$$= \frac{a}{\beta}\left(T_c - \frac{2\gamma}{a}\right) - \frac{a}{\beta}T$$

$$= \frac{a}{\beta}(T_c^* - T)$$

$T_c^* = T_c - 2\frac{\gamma}{a}$, T_c will reduced

(assuming both A and B are $\propto (T_c - T)$)