

Physics 140B: Problem Set 7

5/25/05. Due 6/1/05.

May 25, 2005

1. Superconducting Gap. 30 Points. Make an estimate of the coherence of a superconductor, using the following heuristic approach. A “planewave” like SC wavefunction $\Psi(z) = \Psi_0 \exp(ikz)$ has a constant density $|\Psi_0|^2$. A state whose density varies as $[1 + \cos(qz)]|\Psi_0|^2$ is given by

$$\Phi(z) = \frac{1}{\sqrt{2}}(e^{ikz} + e^{i(k+q)z}).$$

- (a) Show that this wavefunction gives rise to the stated density variation.
(b) Obtain the difference in kinetic energies for $\Psi(z)$ and $\Phi(z)$, where for example

$$K_\Psi = \int dz \Psi(z)^* \frac{p_z^2}{2m} \Psi(z)$$

and the momentum operator is $p_z \equiv -i\hbar \frac{d}{dz}$. The volume of integration is that volume over which the wavefunction is normalized (that is all you need to know.) Treat q as small compared to k and keep on the leading term in q .

(c) **Estimate the coherence length as follows.** Suppose that the difference in energy of Φ relative to Ψ can only be sustained if it is less than or equal to the superconducting energy gap, which we will call 2Δ here. The *coherence length* ξ is the length scale over which the SC density can vary, hence we identify $\xi = 1/q$. Then ξ is identified by setting the difference in kinetic energies to 2Δ . Set k to k_F [the SC electrons live on the Fermi surface] and express it in terms of the Fermi velocity v_F to get the (correct!) expression for ξ .

(d) **Using the correct crystal structure, the correct lattice constant, and the free electron model, evaluate ξ for Cs, Ca, Al, and Pb** (these have valences one, two, three, four respectively).

2. Materials Characteristics. 30 Points. Do all parts of Problem 16.5 of the text (Gersten and Smith), using relationships given in Chapter 16 or elsewhere in the text.