Physics 140B: Problem Set 6

5/18/05. Due 5/25/05.

May 20, 2005

1. Superconducting Condensation Energy. 15 Points. (a) Using Eq. (16.5) in the text, calculate the condensation energy at T=0, in both J/m^3 and eV/electron, for Pb for which $H_{c0} = 6.4\times10^4$ A/m.
(b) Compute your result from part (a) with the expression $\varepsilon(0)\frac{\varepsilon(0)}{E_F}$, where the superconducting gap of Pb is taken to be $2\varepsilon(0) = 2.6$ meV. Note: $\frac{\varepsilon(0)}{E_F}$ gives a rough measure of the fraction of electrons actually affected by the opening of the gap.

2. Critical Current. 20 Points. When a transport current i flows through a superconducting wire of radius R, its path is confined to a region of thickness λ, the penetration depth, just inside the surface of the wire.
(a) In this case show that the critical current density $J_c = i_c A_{eff}$ is independent of R and can be expressed in terms of the critical field H_c by $J_c = H_c/\lambda$. Here A_{eff} is the effective area through which the current flows, with $A_{eff} << \pi R^2$.
(b) Calculate J_c for Pb at T=0. Note: H_{c0} and $\lambda(T = 0)$ are given in Tables 16.1 and 16.4.
(c) Sketch $J_c(T)/J_c(0)$ from T=0 to T$_c$ using the temperature dependencies of H_c and λ given in Eqs. (16.6) and (16.11).

3. Josephson Coupling of Two Superconductors. 25 Points Consider a model of two weakly coupled but identical superconductors, with a coupling of the Josephson type that allows a phase difference between the superconductors. Let $\hat{A} \equiv Ae^{i\phi_A}, \hat{B} \equiv Be^{i\phi_B}$ be the corresponding superconducting complex order parameters, assumed to be constant within each one. Then a Landau-type free energy difference can be written

$$\Delta F = \alpha A^2 + \frac{1}{2}\beta A^4 + \alpha B^2 + \frac{1}{2}\beta B^4 + \gamma(\hat{A}\hat{B}^* + \hat{A}^*\hat{B}).$$

Suppose the coupling parameter γ to be positive.
(a) Minimize the functional with respect to ϕ_A to obtain a substantial simplification of the problem (the phase almost goes away). Express the physical implication of the result in words (even if it is not so obvious physically what the implications are).
(b) Show that there is a solution with $A = B$; recall that these are the *amplitudes* of the order parameters and are non-negative. Obtain the solution for A (or A^2 if you prefer) and find the effect of the coupling on A^2. Reduce the expression to the usual form we're used to seeing.
(c) Evaluate the free energy difference (above) for this solution, and discuss what value(s) of the relative phases $\phi_A - \phi_B$ correspond to the minimum energy.
Extra credit: explore $A \neq B$ solutions for this problem. Interpret what your algebra is indicating.