Physics 140B: Problem Set 5

5/6/05. Due 5/11/05. Problems are 30 points each.

1. **Ordering with Cubic anisotropy.** Consider the free energy functional

   \[ F = F_0 + \frac{1}{2} t \bar{m}^2 + \frac{1}{4} u (\bar{m}^2)^2 + \frac{1}{4} v \sum_j m_j^4, \]

   where \( \bar{m} = (m_1, m_2, m_3) \) is the 3D order parameter (think of magnetism). The last term results in cubic anisotropy. Assume as usual that \( t \) changes sign at \( T_c > 0 \).

   (a) Determine the phase diagram, i.e. for values of the parameters \( u, v \) that are physically allowable, find what direction the magnetization \( \bar{m} \) points in. Consider only the high symmetry directions \((1,0,0), (1,1,0), (1,1,1)\) for the direction of magnetization. Hint: in solving this problem, consider each direction separately from the beginning. Thus for the \((1,1,1)\) direction, \( m_1^2 = m_2^2 = m_3^2 = m^2 / 3 \).

   (b) What are the restrictions on \( u \) and \( v \) for a finite magnetization?

2. **Coupled Order Parameters.** Consider two coupled order parameters A and B (both real scalars) in a Landau free energy, coupled in the simplest way and allowing no inhomogeneity (no gradients of the order parameters):

   \[ F = F_0 + \alpha_1 A^2 + \frac{1}{2} \beta_1 A^4 + \alpha_2 B^2 + \frac{1}{2} \beta_2 B^4 + \gamma A^2 B^2. \]

   Consider that, in the absence of coupling \((\gamma = 0)\) there is an instability to A-type order at \( T_A \) and to B-type order at \( T_B \):

   \[ \alpha_1 = a_1(T - T_A); \quad \alpha_2 = a_2(T - T_B). \]

   Suppose \( T_A \) is the higher temperature, so A-type order is encountered first upon lowering the temperature.

   (a) Minimize the free energy with respect to \( A \) and \( B \), and show that there is a 2nd order transition with \( A^2 = a_1^*(T_A^* - T) \). Find expressions for both the renormalized critical temperature \( T_A^* \) and the renormalized amplitude \( A_1^* \).

   (b) Discuss the effect of the sign of the coupling \( \gamma \) and its magnitude.

   (c) What are the consequences if \( T_A = T_B \)?