Physics 140B: Exam #1

4/22/05.

1. EM Propagation in an Ionic Crystal. 40 points.
You have a 1 meter cube of a single crystalline ionic insulator. Its dielectric properties [i.e. $\epsilon(\omega)$] can be characterized by the transverse optic mode frequency ω_T and longitudinal mode frequency ω_L in the way we have studied. The ionic plasma frequency $\Omega \equiv \Omega_p$ is given, if you want to use it, by $\omega_L^2 = \Omega^2 + \omega_T^2$. For this crystal, $\hbar\omega_L = 50$ meV, $\hbar\omega_T = 30$ meV.

(a) Write the **dielectric function**, and evaluate the **static index of refraction**.

(b) Give the **range of frequencies** for which the crystal *reflects* radiation that is shone on it. (You may express the result in energy, since the ω's are given in energy units.)

(c) A beam of nearly monochromatic radiation is shone on the crystal, its mean frequency corresponds to ω_m=20 meV, and its spread is $\Delta\omega = 1$ meV. The maximum and minimum frequencies in this beam travel at different speeds in the crystal, which you can obtain from part (a), i.e. the index of refraction. Give the **average speed and the difference in speeds** of "light" (infrared, in this case) in this beam. Treat $\Delta\omega/\omega_m$ as a small parameter to simplify your algebra.

(d) When the fastest portion (frequency) of this beam reaches the far edge of the crystal (one meter) **how far behind (in distance)** is the slower part of the beam? Fast/slow here refers to actual speed of travel of the wave, not to the frequency of wiggling.

2. First Order Phase Transitions. 25 points.
The melting of a solid is a first order phase transition.

(a) **Describe the general thermodynamic condition** that defines the melting temperature T_m.

(b) What thermodynamic quantity(s) gives the **latent heat** at the melting transition? Can the latent heat be positive for some materials and negative for others?

3. Intrinsic Semiconductor with 2D Electronic Structure. 20 points.
As you recall from PHY 140A, the density of states N(ε) for a 2D dispersion relation

$$\varepsilon_k = \frac{\hbar^2 \vec{k}^2}{2m^*}; \quad \vec{k} = (k_x, k_y)$$

is constant. Thus for a 2D semiconductor, the valence band $N_h(\varepsilon)$ is a constant below the valence band maximum E_v and the conduction band $N_e(\varepsilon)$ is constant above the conduction band minimum E_c. The band gap then is E_g=E_c - E_v. The *chemical potential* $\mu(T)$ is *defined* such that the number of excited electrons n_e(T) is equal to the number of holes $n_h(T)$. Assume that $k_B T \ll E_g$ and make suitable approximations to simplify your equations and results. Making use of the fact that N(ε) $\propto m^*$ for each band separately, that is, N(ε) = C m^* where the constant C is the same for each band:

(a) **Obtain an expression for** $\mu(T)$ accurate to lowest order in k_BT/E_g. **Interpret** briefly why the dependence on both the T$\rightarrow$value, and the change with T, **has the dependence on the effective masses** that it has.

4. Optical Items. 15 points.
(a) In most ways the three noble metals Cu, Ag, and Au are very similar. **Explain briefly** why Ag looks so different from the other two.

(b) The optical properties of simple metals are very well described by the Drude model for $\epsilon(\omega)$. The simple metals reflect visible light very well, in fact they make reasonable mirrors if their surface is very clean. Explain as simply as you can, **why they reflect** light so well and **what conditions** the constants in the Drude model must satisfy.