

$$M_A \equiv M_1, M_B \equiv M_2$$

3. Let's simplify notation:  $\bar{K} \equiv \frac{1}{2}(K+K')$ ,  $\bar{M} = \frac{1}{2}(M_A+M_B)$  {mean values}

$$\tilde{M} = \sqrt{M_A M_B} = \text{geometric mean. } M_A/M_B = \epsilon \text{ (small); } \Delta M = M_B - M_A$$

$$\omega_{\pm} = A \pm \sqrt{B^2 + C^2 + D^2}, \quad A = 2\bar{K}\bar{M}/\tilde{M}^2, \quad B = \frac{\bar{K}}{\tilde{M}} \frac{\Delta M}{\tilde{M}} = \frac{\bar{K}}{M_A} (1+\epsilon)$$

$$C = -\frac{1}{\tilde{M}}(K+K' \cos ka), \quad D = -\frac{K'}{\tilde{M}} \sin ka.$$

Consider  $ka=0$  (simplest case). Then  $D=0$

and

$$\omega_{\pm} = \frac{2\bar{K}\bar{M}}{\tilde{M}} = \frac{2\bar{K}}{M_A} (1+\epsilon) \approx \frac{2\bar{K}}{M_A} \text{ for } +; \omega_{-} \equiv 0 \text{ (of course)}$$

As  $M_A$  gets small, the upper (optic) frequency gets

large as  $1/M_A$ . Physically, this small mass is coupled to the two large ( $M_A$ ) masses on each side, which cannot vibrate at the same high frequency as the small mass.

Small mass vibrating by itself.

Extra info: if you would look at the zone boundary  $\pi/a$ ,

the  $\omega_{+}(\pi/a)$  frequency would be almost the same: the

frequency is almost dispersionless. Checking the

eigenvectors would show that the heavy mass is hardly

moving at all for the optic mode. The acoustic mode,

on the other hand, is mostly heavy mass motion, with

the light mass just staying halfway between heavy masses.