

Another Way:

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Diffraction condition:  $\Delta \vec{k} = \vec{G}$

$$\Rightarrow |\vec{k}' - \vec{k}| = |\vec{G}|, \quad \vec{G} = \frac{2\pi}{a} \hat{x} + \frac{2\pi}{a} \hat{y} + \frac{2\pi}{a} \hat{z} \quad \text{in this case} \\ \text{(111) plane)}$$

$$k'^2 - 2|\vec{k}'||\vec{k}| \cos 2\theta + k^2 = G^2, \quad |\vec{k}| = |\vec{k}'| = k = \frac{2\pi}{\lambda}$$

$$2k^2 - 2k^2 \cos 2\theta = \frac{(2\pi)^2}{a^2} \times 3$$

$$2k^2 (1 - \cos 2\theta) = 3 \times \frac{(2\pi)^2}{a^2}$$

$$2k^2 \cdot 2 \sin^2 \theta = 3 \times \frac{(2\pi)^2}{a^2}$$

$$\Rightarrow \sin^2 \theta = \frac{3 \times (2\pi)^2}{a^2 \times 4k^2} = \frac{3 \times (2\pi)^2}{4a^2 \times (2\pi)^2 / \lambda^2} = \frac{3\lambda^2}{4a^2}$$

$$\Rightarrow \theta = \sin^{-1} \left( \frac{\sqrt{3}\lambda}{2a} \right) = \sin^{-1} \left( \frac{\sqrt{3} \times 1.54}{2 \times 5} \right) \approx 15.5^\circ$$

The angle between  $\vec{k}$  and  $\vec{k}' = 2\theta \approx 31^\circ$