2. Structure factor : $S_G = \sum_j^{\text{basis}} f_j \exp(-i \vec{G} \cdot \vec{r}_j)$

$\uparrow$ atomic form factor

Reciprocal lattice $\vec{G} = \frac{2\pi}{a}(v_1 \hat{i} + v_2 \hat{j} + v_3 \hat{k})$ for s.c. lattice

(a) diamond

Use simple cubic as the conventional cell.

The basis consists of eight atoms with positions at

$(0,0,0)$, $a(\frac{1}{2},\frac{1}{2},0)$, $a(\frac{1}{2},0,\frac{1}{2})$, $a(0,\frac{1}{2},\frac{1}{2})$,

$a(\frac{1}{4},\frac{1}{4},\frac{1}{4})$, $a(\frac{3}{4},\frac{3}{4},\frac{1}{4})$, $a(\frac{3}{4},\frac{1}{4},\frac{3}{4})$, $a(\frac{1}{4},\frac{3}{4},\frac{3}{4})$.

$$\Rightarrow S_G = f\Big[1 + e^{-i\pi(v_1+v_2)} + e^{-i\pi(v_1+v_3)} + e^{-i\pi(v_2+v_3)} + e^{-\frac{i\pi}{2}(v_1+v_2+v_3)} + e^{-\frac{i\pi}{2}(3v_1+3v_2+v_3)} + e^{-\frac{i\pi}{2}(3v_1+v_2+3v_3)} + e^{-\frac{i\pi}{2}(v_1+3v_2+3v_3)}\Big]$$

$$= f\left[1 + e^{-i\pi(v_1+v_2)} + e^{-i\pi(v_1+v_3)} + e^{-i\pi(v_2+v_3)}\right] \times \left[1 + e^{-\frac{i\pi}{2}(v_1+v_2+v_3)}\right]$$

$$= S_G(fcc)\left[1 + e^{-\frac{i\pi}{2}(v_1+v_2+v_3)}\right]$$

as in (b).

Where $S_G(fcc) = f\left[1 + e^{-i\pi(v_1+v_2)} + e^{-i\pi(v_1+v_3)} + e^{-i\pi(v_2+v_3)}\right]$