PHYS 140A HW 3

1\. The functions are all of the form $\sin(mGx) = \frac{e^{imGx} - e^{-imGx}}{2i}$ or cosine, with $G = \frac{2\pi}{a}$ being the reciprocal lattice vector. As a matter of terminology, e^{imGx} is called the "m^{th} Fourier component (or function)".

$$A\sin\left(\frac{2\pi x}{a}\right) = A\,\frac{e^{iGx} - e^{-iGx}}{2i} = \frac{-A}{2} i\, e^{iGx} + \frac{A}{2} i\, e^{-iGx}$$

$$B\cos\left(\frac{4\pi x}{a}\right) = B\,\frac{e^{i2Gx} + e^{-i2Gx}}{2} = \frac{B}{2} e^{i2Gx} + \frac{B}{2} e^{-i2Gx}$$

$$C\sin\left(\frac{10\pi x}{a}\right)\cos\left(\frac{12\pi x}{a}\right) = C\,\frac{e^{i5Gx} - e^{-i5Gx}}{2i} \times \frac{e^{i6Gx} + e^{-i6Gx}}{2}$$

$$= C\left(-\frac{i}{4}\right)\left\{ e^{i11Gx} + e^{-iGx} - e^{iGx} - e^{-i11Gx} \right\}$$

$$= -\frac{i}{4} C e^{i11Gx} - \frac{i}{4} C e^{-iGx} + \frac{i}{4} C e^{iGx} + \frac{i}{4} C e^{-i11Gx}$$

So looking at all terms, the Fourier components are:

$m = 1$: $-\frac{i}{2}A - \frac{i}{4}C = -\frac{i}{4}(2A + C)$

$m = -1$: $\frac{i}{2}A + \frac{i}{4}C = \frac{i}{4}(2A + C)$

$m = 2$: $\frac{B}{2}$

$m = -2$: $\frac{B}{2}$

$m = 11$: $-\frac{i}{4}C$

$m = -11$: $\frac{i}{4}C$

All others are zero, by inspection.