$$\vec{R}_1 \times \vec{R}_2 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix} \times \frac{a^2}{4} = (0,-2,-2)\frac{a^2}{4}$$

Reciprocal lattice vectors :

$$\vec{G}_1 = 2\pi \frac{\vec{R}_2 \times \vec{R}_3}{\vec{R}_1 \cdot (\vec{R}_2 \times \vec{R}_3)} = 2\pi \frac{(-2,-2,0)\frac{a^2}{4}}{-a^3/2} = \frac{2\pi}{a}(1,1,0)$$

$$\vec{G}_2 = 2\pi \frac{\vec{R}_3 \times \vec{R}_1}{\vec{R}_1 \cdot (\vec{R}_2 \times \vec{R}_3)} = 2\pi \frac{(-2,0,-2)\frac{a^2}{4}}{-a^3/2} = \frac{2\pi}{a}(1,0,1)$$

$$\vec{G}_3 = 2\pi \frac{\vec{R}_1 \times \vec{R}_2}{\vec{R}_1 \cdot (\vec{R}_2 \times \vec{R}_3)} = 2\pi \frac{(0,-2,-2)\frac{a^2}{4}}{-a^3/2} = \frac{2\pi}{a}(0,1,1)$$

(b) bcc lattice vectors :

$$\begin{cases} \vec{R}_1 = (1,1,-1)\frac{a}{2} \\ \vec{R}_2 = (1,-1,1)\frac{a}{2} \\ \vec{R}_3 = (-1,1,1)\frac{a}{2} \end{cases}$$

bct lattice vectors :

$$\begin{cases} \vec{R}_1' = (1,0,0)a \\ \vec{R}_2' = (0,1,0)a \\ \vec{R}_3' = (1,1,1)\frac{a}{2} \end{cases}$$

We can use a set of lattice vectors to represent all the points of this lattice.

(i)

$$\begin{cases} \vec{R}_3' = \frac{a}{2}(1,1,1) = \vec{R}_1 + \vec{R}_2 + \vec{R}_3 \\ \vec{R}_2' = a\,(0,1,0) = \vec{R}_1 + \vec{R}_3 \\ \vec{R}_1' = a\,(1,0,0) = \vec{R}_1 + \vec{R}_2 \end{cases}$$

$\Rightarrow$ $\vec{R}_1'$. $\vec{R}_2'$ and $\vec{R}_3'$ can be represented as linear combinations of $\vec{R}_1$. $\vec{R}_2$ and $\vec{R}_3$.