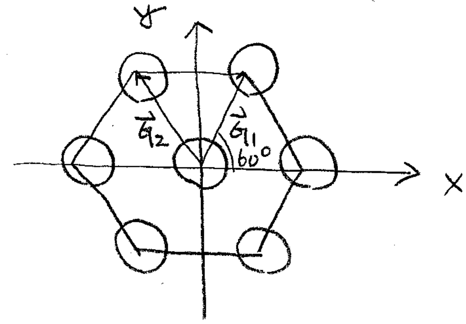
$$\vec{G}_2 = 2\pi \frac{\vec{R}_3 \times \vec{R}_1}{\vec{R}_1 \cdot (\vec{R}_2 \times \vec{R}_3)} = 2\pi \frac{(-\frac{1}{2}ac)\hat{x} + (\frac{\sqrt{3}}{2}ac)\hat{y}}{\frac{\sqrt{3}}{2}a^2 c} = \frac{-2\pi}{\sqrt{3}a}\hat{x} + \frac{2\pi}{a}\hat{y}$$

$$\vec{G}_3 = 2\pi \frac{\vec{R}_1 \times \vec{R}_2}{\vec{R}_1 \cdot (\vec{R}_2 \times \vec{R}_3)} = 2\pi \frac{(\frac{\sqrt{3}}{2}a^2)\hat{z}}{\frac{\sqrt{3}}{2}a^2 c} = \frac{2\pi}{c}\hat{z}$$

$$\Rightarrow \begin{cases} \vec{G}_1 = \frac{4\pi}{\sqrt{3}a}\left(\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}\right) \\ \vec{G}_2 = \frac{4\pi}{\sqrt{3}a}\left(-\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}\right) \\ \vec{G}_3 = \frac{2\pi}{c}(\hat{z}) \end{cases}$$

This is also a hexgonal lattice with a rotation of axis !



Aspect ratio remains the same. Thus :

$$\frac{C_{\text{reciprocal}}}{A_{\text{reciprocal}}} = \frac{\frac{2\pi}{c}}{\frac{4\pi}{\sqrt{3}a}} = \frac{c}{a}$$

$$\Rightarrow \frac{c^2}{a^2} = \frac{\sqrt{3}}{2}, \quad \frac{c}{a} = \sqrt{\frac{\sqrt{3}}{2}} \sim 0.931$$

3. $\vec{R}_1 = (1, 1, -1)\frac{a}{2}, \quad \vec{R}_2 = (1, -1, 1)\frac{a}{2}, \quad \vec{R}_3 = (-1, 1, 1)\frac{a}{2}$

(a) $$\vec{R}_1 \cdot (\vec{R}_2 \times \vec{R}_3) = \frac{a^3}{8}\begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = \frac{a^3}{8} \times (-3+1-2) = -\frac{a^3}{2}$$

$$\vec{R}_2 \times \vec{R}_3 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix} \times \frac{a^2}{4} = (-2, -2, 0)\frac{a^2}{4}$$

$$\vec{R}_3 \times \vec{R}_1 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -1 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} \times \frac{a^2}{4} = (-2, 0, -2)\frac{a^2}{4}$$