Physics 140A HW Set 2

1.
$$\vec{G}_1 = 2\pi \frac{\vec{R}_2 \times \vec{R}_3}{\vec{R}_1 \cdot (\vec{R}_2 \times \vec{R}_3)}$$
$$\vec{G}_2 = 2\pi \frac{\vec{R}_3 \times \vec{R}_1}{\vec{R}_1 \cdot (\vec{R}_2 \times \vec{R}_3)}$$
$$\vec{G}_3 = 2\pi \frac{\vec{R}_1 \times \vec{R}_2}{\vec{R}_1 \cdot (\vec{R}_2 \times \vec{R}_3)}$$

Let the reciprocal lattice vectors of $\vec{G}_1$. $\vec{G}_2$ and $\vec{G}_3$ be $\vec{V}_1$. $\vec{V}_2$ and $\vec{V}_3$. Thus:

$$\vec{V}_1 = 2\pi \frac{\vec{G}_2 \times \vec{G}_3}{\vec{G}_1 \cdot (\vec{G}_2 \times \vec{G}_3)}$$

$$= 2\pi \times \frac{(2\pi)^2 \dfrac{(\vec{R}_3 \times \vec{R}_1) \times (\vec{R}_1 \times \vec{R}_2)}{[\vec{R}_1 \cdot (\vec{R}_2 \times \vec{R}_3)]^2}}{(2\pi)^3 \dfrac{(\vec{R}_2 \times \vec{R}_3) \cdot [(\vec{R}_3 \times \vec{R}_1) \times (\vec{R}_1 \times \vec{R}_2)]}{[\vec{R}_1 \cdot (\vec{R}_2 \times \vec{R}_3)]^3}}$$

$$= \frac{\vec{R}_1 \cdot (\vec{R}_2 \times \vec{R}_3)\ (\vec{R}_3 \times \vec{R}_1) \times (\vec{R}_1 \times \vec{R}_2)}{(\vec{R}_2 \times \vec{R}_3) \cdot (\vec{R}_3 \times \vec{R}_1) \times (\vec{R}_1 \times \vec{R}_2)}$$

Use $(\vec{c} \times \vec{a}) \times (\vec{a} \times \vec{b}) = (\vec{c} \cdot \vec{a} \times \vec{b})\vec{a}$

$$= \frac{\vec{R}_1 \cdot (\vec{R}_2 \times \vec{R}_3)\ \vec{R}_1\ (\vec{R}_3 \cdot \vec{R}_1 \times \vec{R}_2)}{(\vec{R}_2 \times \vec{R}_3) \cdot \vec{R}_1\ (\vec{R}_3 \cdot \vec{R}_1 \times \vec{R}_2)} = \vec{R}_1$$

By the same way, we can get $\vec{V}_2 = \vec{R}_2$ and $\vec{V}_3 = \vec{R}_3$

So the reciprocal lattice of the reciprocal lattice is the original lattice, i.e., $\vec{G}_1$. $\vec{G}_2$ and $\vec{G}_3$ are truly reciprocal lattice of $\vec{R}_1$. $\vec{R}_2$ and $\vec{R}_3$ and vice versa.