Physics 140A: Homework Problem Set 5

Assigned 2/3/05. Due 2/10/05.

1. Phonon Density of States. 20 points.
A phonon dispersion curve that is more realistic than the Debye model is given by

$$\omega(k) = c|k| - \frac{1}{2}d|k|^2, \quad d = \frac{ca}{\pi}.$$

Here c is the speed of sound (a constant that is given) and d is chosen so the dispersion curve is flat at $|k| = \pi/a$.
(a) Calculate the phonon velocity $v(k)$ and confirm that it is zero at $k = \pi/a$.
(b) In preparation for calculation of the density of states $\rho(\omega)$, calculate the wavevector $k(\omega)$ that satisfies $\omega - \omega(k)$=0. There are two solutions to the equation that arises, choose the correct one and describe why it is the correct one.
(c) Evaluate the density of states $\rho(\omega)$ in one dimension. Provide a sketch of the result.
(c) Evaluate the density of states in two dimensions, taking a spherical approximation for the Brillouin zone with boundary at $|\vec{k}| = \pi/a$. Sketch this result also, taking care to draw the $\omega \to 0$ limiting behavior correctly (show why analytically).

2. Dirac Delta Function. 5 points.
From the two basic properties of the δ-function

$$\int \delta(x)dx = 1; \quad \delta(x) = 0, x \neq 0$$

show that

$$\delta(ax) = \frac{1}{|a|}\delta(x).$$

3. Einstein Model. 5 points.
Use the "Einstein model" of N identical non-interacting oscillators (which corresponds to an optic phonon branch without dispersion: $\omega(k) = \omega_E$, a constant) to calculate the specific heat.

4. Debye System in 2 Dimensions. 10 points.
Use the Debye model to calculate the specific heat of a 2D crystal. In doing this, determine first the "Debye wavevector" that contains the same number of states as the Brillouin zone. Compare results with the 3D result in the text.