

Large orbital moment and spin-orbit enabled Mott transition in the Ising Fe honeycomb lattice

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BaFe₂(PO₄)₂ is an unusual Ising insulating ferromagnet based on the Fe²⁺ spin S=2 ion, whose susceptibility suggests in addition a large orbital component to the Fe local moment. We apply density functional theory based methods to obtain a microscopic picture of the competing interactions and the critical role of spin-orbit coupling (SOC) in this honeycomb lattice system. The low *T* ferromagnetic phase displays a half-semimetallic Dirac point pinning the Fermi level and preventing gap opening before consideration of SOC, presenting a case in which correlation effects modeled by a repulsive Hubbard *U* fails to open a gap. Simultaneous inclusion of both correlation and SOC drives a large orbital moment M_L of more than $0.7 \mu_B$ (essentially $L = 1$) for spin aligned along the \hat{c} -axis, with a gap comparable with the inferred experimental value. The large orbital moment accounts for the large Ising anisotropy, in spite of the small magnitude of the SOC strength on the 3*d* (Fe) ion. Ultimately, the Mott-Hubbard gap is enabled by degeneracy lifting by SOC and the large Fe moments, rather than by standard Hubbard interactions alone. We suggest that competing orbital occupations are responsible for the structural transitions involved in the observed re-entrant rhombohedral-triclinic-rhombohedral sequence.

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I. INTRODUCTION

In the last few decades, the two-dimensional (2D) honeycomb lattice has attracted interest due to several exotic physical phenomena related with superconductivity, magnetism, and topological phases, and to the

Dirac point 2D material graphene. Recently, discoveries of topological insulators, originally on the honeycomb lattice,[1] have stimulated increased research on properties of systems with honeycomb lattices.[2–4]

Transition metal d^1 systems are good examples to investigate the interplay among lattice, spin, orbital degrees of freedom, and correlation effects, since variation of $3d \rightarrow 4d \rightarrow 5d$ leads to weakening strength of correlation, but strengthening of spin-orbit coupling (SOC). For $5d^1$ systems, a large SOC results in a $J_{eff} = \frac{3}{2}$ Dirac (or relativistic) Mott insulator in Ba₂NaOsO₆[5, 6] in which the orbital moment plays a pivotal role. Similarly, a $J_{eff} = \frac{1}{2}$ Mott transition has been proposed in $5d^5$ Sr₂IrO₄, *i.e.* one hole in the t_{2g} manifold.[7] $4d^1$ systems usually have a reduced SOC and some enhanced correlation strengths. In Nb₁₂O₂₉,[8] a highly *localized* d^1 Nb⁵⁺ ion coupled to *itinerant* Nb⁴⁺ carriers induces a new two dimensional Kondo-Heisenberg lattice system. In $3d$ systems, a much stronger correlation strength leads to a Mott transition, which has been intensively discussed,[9] while SOC is usually minor. Here, we focus on the recently synthesized BaFe₂(PO₄)₂ compound with a honeycomb lattice of high spin d^6 Fe²⁺ ions, leading to an effectively isolated minority spin d^1 configuration due to a large exchange splitting of the high spin Fe ion. Large $3d$ orbital moments have been encountered recently[10] but understanding their origin is not simple.

Mentré and coworkers synthesized a rare example of an insulating 2D Ising ferromagnetic (FM) oxide BaFe₂(PO₄)₂, confirmed by the observed critical exponents in good agreement with the theoretical Ising values, with the Curie temperature $T_C = 65$ K and negligible hysteresis.[11, 12] This system also shows an unusual

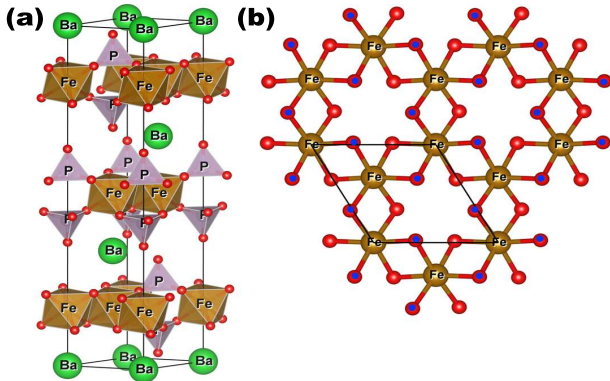


FIG. 1: (Color online) (a) The rhombohedral structure, which contains PO₄ tetrahedra and edge-sharing FeO₆ octahedra, of BaFe₂(PO₄)₂ in its low and high *T* regimes. (b) Top view of the 2-dimensional Fe honeycomb lattice in the $\hat{a} - \hat{b}$ plane. The solid line indicates the unit cell, containing two Fe ions (brown circles). The oxygen ions (small red circles) in the upper layer are denoted by a dot.

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TABLE I: Optimized internal parameters and structure information with the experimental lattice parameters of $a=4.869$ Å and $c=23.230$ Å at low T . [12] In the representation of a hexagonal lattice, the Ba ions are at $3a$ (0,0,0). The Fe, O1, and P atoms sit $6c$ (0,0, z), while O2 lies at $18f$ (x , y , z). The O2 ions are corner-shared by the PO₄ tetrahedra and the FeO₆ octahedra, whereas the O1 ions are at one of the vertices of the PO₄ tetrahedra. The bond length d and the bond angles are given in units of Å and degree, respectively.

	Fe	P	O1	O2		
internal	z	z	z	x	y	z
parameter	0.1680	0.5710	0.6371	0.3494	0.0235	0.8840
$d(\text{Fe-O})(\times 3)$				2.042, 2.050		
$d(\text{P-O})$				1.536, 1.598($\times 3$)		
$\angle\text{O-Fe-O}(\times 3)$				86.6, 88.6, 93.2, 91.8		
$\angle\text{O-P-O}(\times 3)$				107.2, 111.7		

re-entrant structural transition sequence: a re-entrant rhombohedral ($R\bar{3}$) \rightarrow triclinic ($P\bar{1}$ at 140 K) \rightarrow rhombohedral ferromagnetic (FM) ($R\bar{3}$ at 70 K). This rare re-entrant sequence was suggested to arise from competition between Jahn-teller distortion and magnetism. [11, 12] The observed Curie-Weiss moment, $6.16 \mu_B$, is substantially enhanced from the spin-only value of $4.9 \mu_B$ for a $S = 2$ system. Additionally, the saturated moment is about $5 \mu_B$ from powder neutron diffraction studies. [11, 12] These values are close to those of the case of $S = 2$, $L = 1$, thus requiring a large orbital moment on the $3d$ ion. The latter corresponds to the possible largest orbital moment $M_L = 1 \mu_B$ for the t_{2g}^1 system. [5] Optical spectroscopy measurements show semiconducting behavior with an estimated energy gap of 1.5 eV. [13]

Although the generalized gradient approximation (GGA) plus on-site Coulomb repulsion U calculations by Mentré and coworkers provided some preliminary results, [11] available information about the electronic structures is still limited. Here we report more extensive density functional theory (DFT) based studies, including correlation and SOC, to uncover the origin of the strong magnetocrystalline anisotropy, Mott transition, and the large orbital moment in BaFe₂(PO₄)₂.

II. CRYSTAL STRUCTURE AND METHODS

Figure 1 shows the rhombohedral structure of the space group $R\bar{3}$ (No. 148), which is observed in the both low and high T regimes. In this structure, P ions sit in the center of the Fe honeycomb lattice, but above and below the layer along the \hat{c} direction. The P ions are edge-shared with three oxygen ions of the FeO₆ octahedra, and form PO₄ tetrahedra. The Fe²⁺ layers are isolated by the intervening insulating (PO₄)³⁻ tetrahedra and Ba²⁺ ions. In the low T phase, the interlayer distance is 7.71 Å along the c -axis, while the Fe-Fe distance is 2.81 Å.

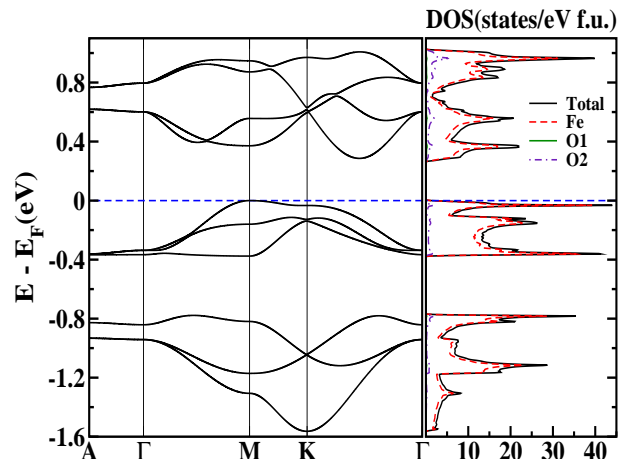


FIG. 2: (Color online) Nonmagnetic GGA band structure (left) and densities of states (right) of the rhombohedral structure in the energy region containing Fe d orbitals. The occupation confirms the d^6 Fe²⁺ configuration and the density of states just below the gap shows a quasi-one-dimensional character due to the flat band that extends from M to K, hence all around the edge of the Brillouin zone. The bottom of the gap is denoted by the horizontal dashed line.

Both structurally and electronically (below), the compound is strongly 2D in nature.

Using the GGA exchange-correlation functional, the internal parameters [14, 15] were optimized at the experimental lattice parameters for the low T phase (see Table I), since no experimental data is available. The optimized positions are very similar to the experimental ones at high T , as expected since the difference in volumes between the low and high T phases are only several tenths of a percent. [11, 12] The FeO₆ octahedra are somewhat more regular in the low T phase, while the PO₄ tetrahedra are close to an ideal tetrahedron in the high T phase.

Our calculations were performed in a rhombohedral unit cell with GGA exchange-correlation functional plus U and SOC, which are implemented in the accurate all-electron full-potential code WIEN2K. [16] On-site Coulomb repulsion $U = 3 - 5$ eV and the Hund's exchange parameter $J = 0.70$ eV were employed, as widely used for Fe²⁺ systems. [11, 17] The basis size was determined from the smallest atomic sphere by $R_{mt}K_{max} = 7$ and augmented plane wave radii of Ba 2.30, Fe 2.00, P 1.61, and O 1.29, in atomic units. The Brillouin zone was sampled up to a k -mesh of $17 \times 17 \times 17$ to check convergence carefully in this system, which is semimetallic at some levels of the theory.

III. RESULTS

A. Underlying electronic structure

Before discussing the low temperature FM phase results, we address the nonmagnetic state, which repre-

sents the underlying electronic structure. The enlarged band structure in the energy range of Fe d orbitals and the corresponding densities of states (DOSs) are given in Fig. 2. In the region displayed there are 10 d bands, since the unit cell contains 2 Fe ions. From the tiny oxygen character in the DOS around E_F , we conclude that direct Fe $t_{2g} - t_{2g}$ hopping is substantial due to the edge-sharing octahedral structure, and this interaction leads to a bonding-antibonding splitting ~ 1 eV of the fully filled t_{2g} manifold. A gap of 0.25 eV opens between the antibonding t_{2g}^* state and the unfilled e_g manifold. This feature of isolated bands around E_F in a hexagonal lattice system is similar to that of the hole-doped superconductor $\text{Li}_{1-x}\text{NbO}_2$. [18, 19] In both bonding and antibonding manifolds, Dirac points appear at the K point by symmetry, as observed in several honeycomb lattices.

An anomalous feature is that the top of the t_{2g}^* manifold (bottom of the gap) is flat along the $M - K$ line, hence along the *entire edge* of the Brillouin zone. Considering also the dispersionless character along the \hat{c} -axis due to the large interlayer distance, the electronic structure becomes an unusual quasi-one-dimensional one, as evident in DOS near E_F . This one-dimensionality however is not uniaxial but instead radial, perpendicular to the edge of the Brillouin zone. A similar one-dimensionality was found as well in the hexagonal lattice CuAlO_3 compound. [20]

B. Ferromagnetic electronic structure

1. Uncorrelated electronic structure and magnetism

Now we focus on the FM state, which is the ground state at the low T regime. The magnetic energy is large, with the FM state being 1.2 eV/Fe lower in energy than for the nonmagnetic state. This very large magnetization energy guarantees that the local moment is robust even in the high T disordered regime. Based on the Stoner picture (which may be accurate only for small moments) where this energy is $\frac{1}{4}IM_S^2$, $M_S = 4\mu_B$ leads to a broad first estimate of the Stoner I of 0.3 eV. A more realistic means of estimating I is from the band exchange splitting $IM_S = \Delta_{ex} = 3$ eV of the GGA bands, giving $I = 0.75$ eV, more in line with accepted values for the magnetic $3d$ ions. Consistent with the experiments, the FM state has the lower energy (in GGA) of 370 meV/Fe than simple antiferromagnetic state.

The left side of Fig. 3(a) displays the GGA FM band structure, showing a half-semimetallic character with a minority spin Dirac point (or Weyl node) [21] lying right below E_F by 30 meV at the point K . The corresponding DOS, given in the right side of Fig. 3(a), shows a $\frac{2}{3}$ -filled minority t_{2g} bonding manifold, confirming the high spin $S=2$ Fe^{2+} configuration. The t_{2g} manifold shows a large exchange splitting of 3 eV. All majority states (t_{2g} and e_g) are filled, hence $\frac{5}{2}$ spin, and orbitally inert. Henceforward we focus on the single minority electron, whose

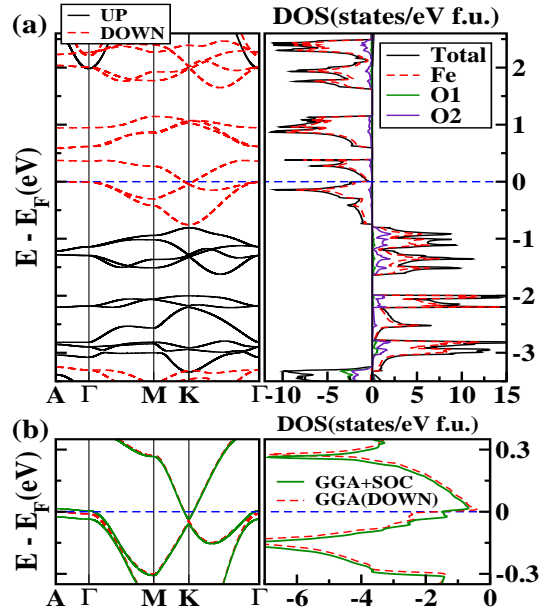


FIG. 3: (Color online)(a) FM band structures and DOSs within GGA in the range of Fe d bands, indicating a half-semimetallic character in the minority bands (red dashed lines). A Dirac point appears almost exactly at E_F at the point K . (b) Overlaid FM GGA (red dashed lines) and GGA+SOC (green solid lines) band structures and total DOS enlarged near E_F , in a ± 0.3 eV region containing only some of the minority t_{2g} manifold. The Dirac point is split by about 30 meV by SOC. Note that a tiny hole pocket compensating a tiny electron pocket at the K point appears along an uncommon symmetry line (invisible here).

spin-half reduces the total spin to $S=2$ and introduces the question of orbital occupation.

With the local X, Y, Z coordinates of the FeO_6 octahedron, the t_{2g} orbitals with global z -axis along \hat{c} can be described in a 3-fold symmetry adapted manner by

$$\phi_m = \frac{1}{\sqrt{3}}(\xi_m^0 d_{XY} + \xi_m^1 d_{YZ} + \xi_m^2 d_{ZX}), \quad (1)$$

where the phase factor is $\xi_m = \exp(i\frac{2\pi m}{3})$ and $m = L_z$ is the projection of the orbital moment. Due to the small trigonal distortion, the $L_z = 0$ orbital ϕ_0 (d_{z^2} in shape) has a somewhat higher site energy than the doublet $\phi_{\pm 1}$. The corresponding DOS near E_F has some similarity to a particle-hole asymmetric version of graphene, [22] due to a Dirac point at K that lies very near the Fermi level. The strong magnetism completely changes the Fe ion configuration from $\{t_{2g}^6 e_g^0\}$ to $\{t_{2g}^{3\uparrow} e_g^{2\uparrow} t_{2g}^{1\downarrow}\}$.

2. Dirac point; semimetal-insulator transition

We now address the observed insulating character of $\text{BaFe}_2(\text{PO}_4)_2$, using the GGA+U approach for correlated insulators. Applying a small $U = 1$ eV with $J = 0.5 - 1$ eV to the Fe ions within GGA+U, the Dirac point

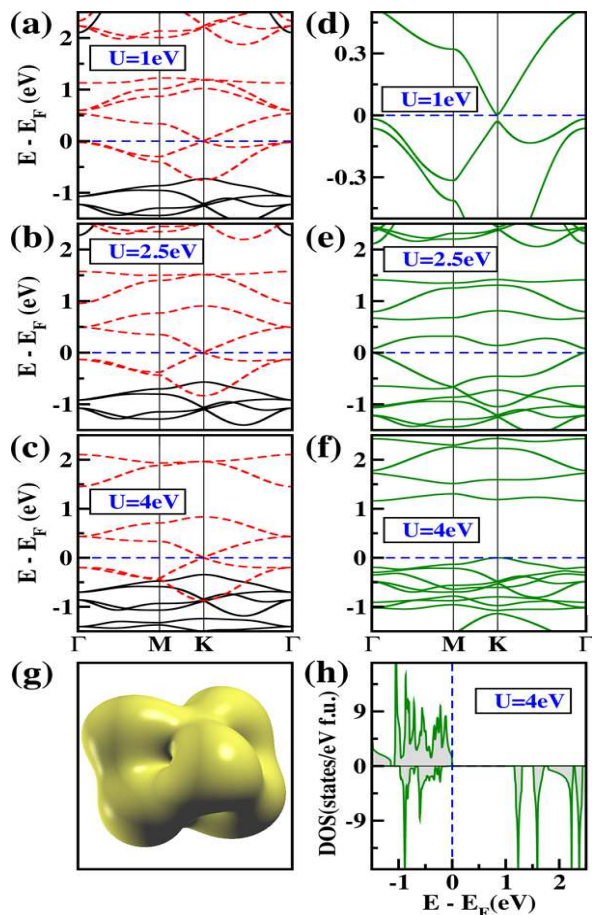


FIG. 4: (Color online) Band structures near E_F : (a)-(c) within GGA+U, and (d)-(f) from GGA+U+SOC, for $U = 1 - 4$ eV and $J = 0.7$ eV. The spin up and down bands in (a)-(c) are denoted by black solid and red dashed lines, respectively, while the bands including SOC (d)-(f) are shown by green solid lines. In (d) the scale is enlarged to allow the gap opening at the Dirac point to be seen. (g) Charge density plot of the minority d orbitals, shown at isovalue of $0.09 e/\text{\AA}^3$. The c -axis is along the vertical direction. This shape is nearly identical for all calculations performed here. (h) total DOS at $U = 4$ eV in GGA+SOC+U, showing a 1.2 eV energy gap between the corresponding up and down bands.

degeneracy pins E_F and no gap opens, as shown in Fig. 4(a). The Dirac cone is isotropic, as in graphene. As U is increased, the Dirac point degeneracy continues to pin E_F . Applying U up to 7 eV, beyond the limit of a proper value for Fe ions, although higher energy bands shift this system remains a half-semimetal without gap, as illustrated in Fig. 4(b) and (c).

As mentioned in the Introduction, experiment shows semiconducting behavior and implies a large orbital moment M_L , [11, 12] which are not explained by GGA+U. We carried out calculations including SOC, both GGA+SOC and GGA+U+SOC. The GGA+SOC band structure, overlapping the GGA one, is shown in the left panel of Fig. 3(b). SOC leads to a band splitting

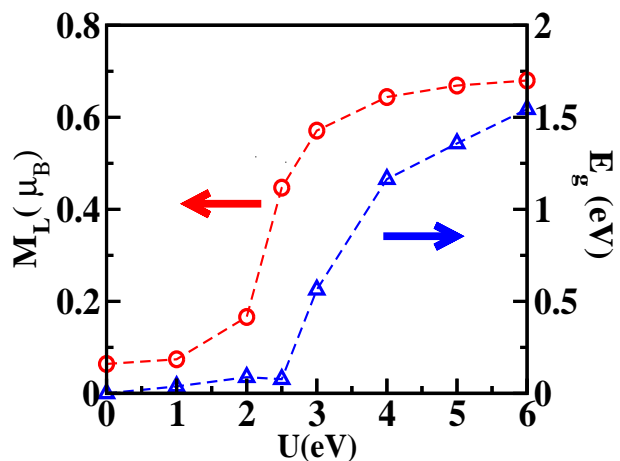


FIG. 5: (Color online) Variation of the Fe orbital M_L moment, and energy gap E_g , varying U in GGA+U+SOC from 0-6 eV, with the spin directed along the \hat{c} -axis. From $U = 4$ eV, the gap is between the filled spin-up and the unfilled spin-down channels. The critical value of U is in the 2.5-3 eV range.

of 40 meV near E_F , visible in this plot only at the Γ and K points. The linear Dirac bands are also split by 30 meV. This small effect is consistent with the small SOC strength in 3d ions, and spin mixing is also degraded by the 2.5 eV exchange splitting.

Another interesting feature occurs in the parabolic bands touching E_F at the Γ point. Inclusion of SOC splits the bands, degenerate at the Γ points, and leaves only one maximum at E_F , which is more isotropic than in the case excluding SOC. As a result, as shown in the DOS of the right side of Fig. 3(b), a van Hove singularity (vHs), much sharper than in GGA, appears very near E_F , in addition to two additional vHs at -0.15 and 0.25 eV.

Simultaneous inclusion of correlation and SOC effects splits the Dirac point degeneracy, thereby opening a gap for U as small as 1 eV. At small U this gap is a spin-orbit gap, not a Mott gap. Although the strength of SOC is small in 3d systems, the symmetry lowering due to SOC becomes crucial for opening a gap in this system, as occurs also in BaCrO_3 . [23] The resulting band structures in GGA+U+SOC are given in Fig. 4(d)-(f). As displayed in Fig. 5, the energy gap of 1.2 eV for $U = 4$ eV is close to the experimentally estimated value. The gap is indirect, between K and Γ , although due to the small dispersion the direct gap is not much larger. There has been some uncertainty in determining the optical gap spectroscopically. [13]

3. Degeneracy lifting, large orbital moment

GGA+SOC gives a negligible magnetocrystalline anisotropy (MCA), with an orbital moment of $M_L \approx 0.08\mu_B/\text{Fe}$ both for in-plane and \hat{c} -axis spin alignment. Varying U in GGA+U+SOC with the spin along \hat{c} , M_L

of Fe rapidly increases in the $U=2-3$ eV range, then saturates near $0.7\mu_B$ at $U = 4$ eV, as displayed in Fig. 5. This value excludes contributions from the tails of the $3d$ orbitals extending out of the Fe sphere, so the atomic value will be somewhat larger. As a critical value U_c of 2.5-3 eV, the opening of a Mott gap begins.

Consistent with the large exchange splitting $\Delta_{ex} = 3$ eV, the total spin moment $M_S = 4\mu_B/\text{Fe}$ remains unchanged, since mixing of the spin-split orbitals by small SOC is minor. At $U = 4$ eV, the spin direction along the \hat{c} direction has much lower energy around 400 meV/f.u. than for spin in the $\hat{a} - \hat{b}$ plane, explaining the observed Ising behavior. The state obtained for the spin direction in the $\hat{a} - \hat{b}$ plane (not pictured) remains semimetallic (Dirac point) even in large U regime, showing a very small M_L of $\sim 0.06 \mu_B$. This striking anisotropy confirms that SOC drives the semimetal-insulator transition, and that depends strongly on spin direction. This type of Ising magnet, *i.e.* with the spin direction perpendicular to the layers, has been proposed as a crucial ingredient for emergence of a quantum anomalous Hall state[4] in ferromagnets.

We now discuss the origin of the large orbital moment. The SOC operator is

$$H_{SO} = \xi \vec{S} \cdot \vec{L} = \xi [S_z L_z + \frac{1}{2}(S_+ L_- + S_- L_+)]. \quad (2)$$

The majority shell is filled, spin-split from the active minority orbital by Δ_{ex} , and inert, because spin-mixing is reduced by $\xi/\Delta_{ex} \sim 10^{-2}$. Thus the spin and orbital operators are those of the minority channel. With the majority channel frozen out, the only effect is from the diagonal term $S_z L_z$ with $S_z = -\frac{1}{2}$. For the magnetization along the \hat{c} axis, the orbital degeneracy is broken by $-\frac{\xi}{2} L_z$ for the two orbital projections. With a bandwidth of 0.5 eV, this small splitting induces an orbital moment, but more importantly it specifies the orbital to be occupied in the Mott insulating phase. For the magnetization in-plane, the $L_z = \pm 1$ degeneracy remains unbroken, no orbital moment arises, and correlation is ineffective in opening a gap, as discussed above.

As can be seen in Fig. 4(g), the spin density plot of the filled minority t_{2g} manifold shows the ϕ_{+1} character characteristic of an $L_z = 1$ orbital. When symmetry allows it, the two orbitals $\phi_{\pm 1}$ are equally filled, the semi-Dirac degeneracy remains at K, so no gap emerges. It is SOC that breaks the degeneracy, which though small, enables the correlation effects to operate. Upon increasing U , the single minority electron density begins to transfer from the ϕ_{-1} orbital to the ϕ_{+1} orbital.

IV. DISCUSSION AND SUMMARY

Using the DFT-based calculations, we have investigated the Ising insulating Fe-honeycomb lattice system $\text{BaFe}_2(\text{PO}_4)_2$, where at the GGA level results in a half-semimetallic electronic structure with a fermionic Dirac

point lying nearly precisely at E_F ; this degeneracy is split by inclusion of SOC. Applying both U and SOC suppresses the direct $t_{2g} - t_{2g}$ interaction, and leads to a Mott transition of uncommon origin, becoming enabled by the symmetry lowering due to the (small in magnitude) SOC. In the insulating phase the magnetic phase is $S = 2, L = 1$. The atomic-like value of L suggests classifying $\text{BaFe}_2(\text{PO}_4)_2$ as a $J_{eff} = 3$ Dirac Mott insulator. These DFT-based studies have explained the large magnetic anisotropy, the very large orbital moment for a $3d$ ion, and the unusual manner in which spin-orbit coupling enables the Mott gap to open in $\text{BaFe}_2(\text{PO}_4)_2$. Even the magnitude of the gap is given reasonably by our methods.

The re-entrant structural transitions mentioned in the Introduction needs comment. The Fe honeycomb lattice studied here bears a great deal of similarity to the (111)-oriented perovskite bilayers that are recently being studied,[24, 25] a difference being that in the latter case the metal-oxygen octahedra are corner sharing rather than edge sharing. In those systems structural symmetry breaking is predicted in some cases, using the same methods as used here. That symmetry breaking involves a new type of orbital-occupation competition. The high symmetry structure (3-fold, as here) supports complex-valued, orbital moment containing orbitals, but breaking to low symmetry (P1 space group in that case) is connected with occupation of the usual (pseudo-)cubic t_{2g} type real orbitals.

The same orbital-occupation competition can be expected in $\text{BaFe}_2(\text{PO}_4)_2$. At high temperature where the Fe spins are disordered (primarily up and down due to the Ising nature) the (space- and time-averaged) symmetry is the high 3-fold symmetry, and the symmetry adapted ϕ_{+1} orbital is occupied and shows up in the Curie-Weiss moment. The ‘‘ground state’’ (with spins still disordered) may be one in which a real-valued t_{2g} orbital is ordered instead, with accompanying symmetry lowering, and the transition toward such a lower symmetry structure occurs at 140 K. However, upon ordering ferromagnetically, the strains that accompany occupation of cubic orbitals (*viz.* Jahn-Teller strains) may be incompatible with long range order. Such spin-lattice competition could then favor a return to the symmetric structure at or near the Curie temperature, as observed. Further consideration of this mechanism will be left for future work.

A Dirac point has played an important role in our discussion of how the Mott gap in $\text{BaFe}_2(\text{PO}_4)_2$ arises only when spin-orbit coupling is included. Few instances of a Dirac point pinned to the Fermi level in a ferromagnet have been reported, which produces a half-semimetallic system. One such instance is in the $\text{CrO}_2/\text{TiO}_2$ multilayer[26] where it is actually a semi-Dirac semi-Weyl point. Another instance is (111)-oriented perovskite superlattices,[25, 27] where Dirac point degeneracies are only lifted by symmetry breaking interactions. Related band points arise in the hexagonal W lattice on the Cl-Si surface,[4] and are also of

interest in triangular lattice systems.[28] In some such systems, Chern bands with a Chern number ± 2 are proposed.[26, 27] In addition, the three-dimensional Weyl semimetals NbP and TaAs, which are distinguished by lack of inversion symmetry but preserving time reversal symmetry, have been very much of interest, since these compounds show an extremely large and unsaturated magnetoresistivity even for very high magnetic field.[29–31] $\text{BaFe}_2(\text{PO}_4)_2$ bears some similarity to the above mentioned compounds, and has the potential for additional interesting properties. It suggests further careful experimental research on the system.

Acknowledgments

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