

# Strong Electron-Phonon Coupling in Yttrium under Pressure

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Linear response methods are applied to identify the increase in electron-phonon coupling in elemental yttrium that is responsible for its high superconducting critical temperature  $T_c$ , which reaches nearly 20 K at 115 GPa. While the evolution of the band structure and density of states is smooth and seemingly modest, there is strong increase in the  $4d$  content of the occupied conduction states under pressure. We find that the transverse mode near the L point of the fcc Brillouin zone, already soft at ambient pressure, becomes unstable (in harmonic approximation) at a relative volume  $V/V_0 = 0.60$  ( $P \approx 42$  GPa). The coupling to transverse branches is relatively strong at all high symmetry zone boundary points X, K, and L. Coupling to the longitudinal branches is not as strong, but extends over more regions of the Brillouin zone and involves higher frequencies. Evaluation of the electron-phonon spectral function  $\alpha^2 F(\omega)$  shows a very strong increase with pressure of coupling in the 2-7 meV range, with a steady increase also in the 7-20 meV range. These results demonstrates strong electron-phonon coupling in this system that can account for the observed range of  $T_c$ .

## I. INTRODUCTION

The remarkable discovery<sup>1</sup> in 2001 of MgB<sub>2</sub> with superconducting critical temperature  $T_c=40$ K, and the fact that the simple free-electron-like metal lithium<sup>2,3,4</sup> also has  $T_c$  in the 14-20K under 30-50 GPa pressure, has greatly increased efforts in seeking higher  $T_c$  in elements and simple compounds. Currently 29 elements are known to be superconducting at ordinary pressure and 23 other elements superconduct only under pressure<sup>5,6</sup>. Among elements there is a clear trend for those with small atomic number  $Z$  to have higher values of  $T_c$ , although much variation exists. For example, under pressure<sup>7,8</sup> Li, B, P, S, Ca, and V all have  $T_c$  in the range 11-20 K. Hydrogen<sup>9,10,11</sup>, the lightest element, is predicted to superconduct at much higher temperature at pressures where it becomes metallic.

While light elements tend to have higher  $T_c$  among elemental superconductors, Hamlin *et al.*<sup>12</sup> recently reported that Y ( $Z=39$ ) superconducts at  $T_c=17$ K under 89 GPa pressure and 19.5 K at 115 GPa, with the trend suggesting higher  $T_c$  at higher pressure. This result illustrates that heavier elements should not be neglected; note that La ( $Z=57$ ) has  $T_c$  up to 13 K under pressure.<sup>13,14</sup> The superconductivity of La has been interpreted in terms of the rapidly increasing density of states of  $4f$  bands near Fermi level with increasing pressure, causing phonon softening and resulting stronger coupling under pressure.<sup>15,16</sup> Such a scenario would not apply to Y, since there are no  $f$  bands on the horizon there. No full calculations of the phonon spectrum and electron-phonon coupling have been carried out for either Y or La to date.

La and Y are two of the few elemental transition metals to have  $T_c$  above<sup>7,8</sup> 10 K, and the case of Y is unusually compelling, since its value of  $T_c$  is at least as high that of Li, qualifying it as having the highest  $T_c$  of any elemental superconductor. Moreover, the reduced volume  $v \equiv V/V_0=0.42$  corresponds to the value of  $T_c \approx 20$ K in Y [12] (115 GPa) and also to the report of  $T_c \approx 20$ K in (strained) Li [2] above 50 GPa.<sup>17,18</sup> For our study of Y reported here, it is first necessary to understand its phase diagram. Under pressure, it follows a structure sequence<sup>19,20</sup> through close-packed phases that is typical of rare earth metals: hcp→Sm-type→dhcp→dfcc (dfcc is distorted fcc, with trigonal symmetry). The transitions occur around 12 GPa, 25 GPa, and 30-35 GPa. Superconductivity was first found<sup>21</sup> in Y by Wittig in the 11-17 GPa range (1.2-2.8 K), in what is now known to be the Sm-type structure. From 33 GPa to 90 GPa  $T_c$  increases smoothly (in fact  $T_c$  increases linearly with decrease in  $v$  over the entire 35-90 GPa range<sup>12</sup>) suggesting that Y remains in the fcc phase, perhaps with the distortion in the dfcc phase vanishing (the tendency is for the  $c/a$  ratio in these structures to approach ideal at high pressure<sup>19</sup>). Calculations<sup>22</sup> predict it adopts the bcc structure at extremely high pressure(>280 GPa), but this is far beyond our interest here.

In this paper we report electronic structure and electron-phonon coupling calculations of Y for reduced volumes in the range  $0.6 \leq v \leq 1$  (pressures up to 42 GPa). Our results indeed show strong electron-phonon coupling and phonon softening with increasing pressure. A lattice instability (in the harmonic approximation used in linear response calculations) is encountered at  $v=0.6$  and persists to higher pres-

tures. The instability arises from the vanishing of the restoring force for transverse displacements for  $Q_{\parallel} < 111 >$  near the zone boundary, corresponding to sliding of neighboring close-packed layers of atoms. It is only the stacking sequence of these layers that distinguishes the various structures in the pressure sequence of structures (see above) that is observed in rare earth metals. Near-vanishing of the restoring force for sliding of these layers is consistent with several stacking sequences being quasi-degenerate, as the structural changes under pressure suggest.

This paper is organized as follows. In Sec. II structural details are given, and the calculational methods are described. Results for the electronic structure and its evolution with pressure are provided in Sec. III. The background for understanding the electron-phonon coupling calculations is provided in Sec. IV, and corresponding results are presented and analyzed in Sec. V. The implications are summarized in Sec. VI.

## II. STRUCTURE AND CALCULATION DETAILS

Yttrium crystallizes in the hcp structure at ambient pressure with space group  $P63/mmc$  (#194) and lattice constants  $a=3.647 \text{ \AA}$  and  $c=5.731 \text{ \AA}$ .<sup>23</sup>

Since the observed structures are all close packed (or small variations from) and above 35 GPa Y is essentially fcc, we reduce the calculational task by using the fcc structure throughout our calculations. The space group is  $Fm3m$  (#225), with the equivalent ambient pressure lattice constant  $a=5.092 \text{ \AA}$ . We do note however that results for electron-phonon strength can be sensitive to the crystal symmetry, both through the density of states and through the nesting function that is described below.

We use the full potential local orbital (FPLO) code<sup>24</sup> to study the electronic structure, and apply the full-potential linear-muffin-tin-orbital (LM-TART) code<sup>25</sup> to calculate the phonon frequencies and the electron-phonon coupling spectral function  $\alpha^2F$ . For FPLO, a  $k$  mesh of  $36^3$  and the Perdew-Wang (PW92)<sup>26</sup> exchange-correlation potential are used. The basis set is  $1s2s2p3s3p3d::(4s4p)/5s5p4d+$ . For LM-TART, a  $k$  mesh of  $48^3$  and GGA96 approximation<sup>27</sup> for exchange-correlation potential are used. For the electron-phonon coupling calculations we used a phonon  $Q$  mesh of  $16^3$ , which has 145  $Q$  points in the irreducible Brillouin zone.

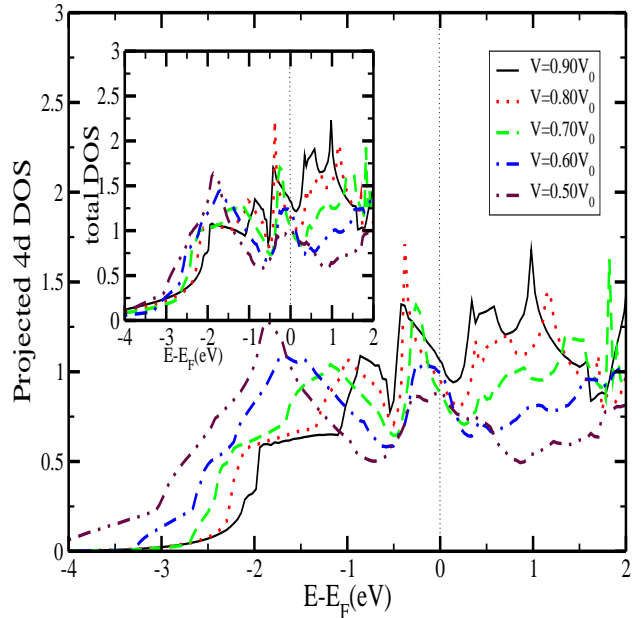


FIG. 1: (Color online) Plot of the total DOS and projected  $4d$  DOS per atom of fcc Y with different volumes. Both the total and the  $4d$  density of states at Fermi level decrease with reduction in volume.

## III. ELECTRONIC STRUCTURE UNDER PRESSURE

Many studies suggest that the general character of an elemental rare earth metal is influenced strongly by the occupation number of the  $d$  electrons, which changes under pressure. Our calculations show that the  $4d$  occupation number of trivalent Y increases from 1.75 at ambient pressure, to a little above 2 at  $V=0.7V_0$  and then finally close to 3 at  $V=0.3V_0$  (which is extreme pressure). Such an increase can be seen from the projected density of states (PDOS) of  $4d$  states (Fig. 1) at different volumes. From Fig. 1 broadening of the density of states with reduction in volume can be seen, but is not a drastic effect. The main occupied  $4d$  PDOS widens from 2 eV to 3 eV with reduction of the volume to  $v=0.5$ . The value of the density of states at the Fermi level (taken as the zero of energy)  $N(0)$  decreases irregularly with volume reduction; the values are given in Table I.

The pressure evolution of the band structure is indicated in Fig. 2, where the  $4d$  character at  $v=1.00$  (black) and  $v=0.50$  (gray) is emphasized. First, the relative positions of the Fermi level crossings change smoothly, indicating there is little change in the Fermi surface topology. This slow change is also

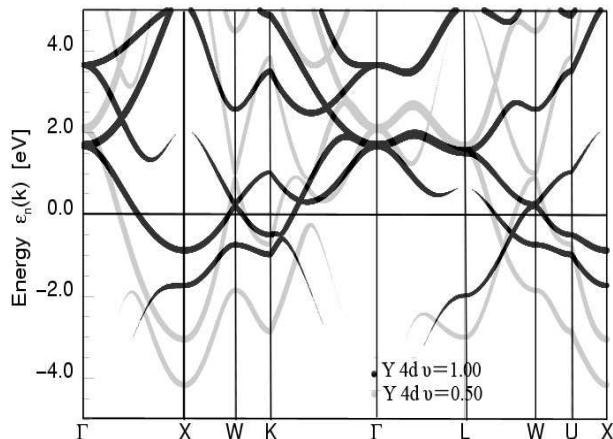


FIG. 2: (Color online) Plot along high symmetry directions of the bands of Y at  $V/V_0=1.00$  and at  $V/V_0=0.50$ . The “fattening” of the bands is proportional to the amount of Y 4d character. Note that the 4d character goes substantially in the occupied bands under pressure (the lighter shading), although there is relatively little change in the Fermi surface band crossings.

seen in Fermi surface plots, of which we show one (below). Second, the overall band widths change moderately, as was noted above in the discussion of the density of states. The change in position of 4d character is more substantial, however. 4d bands at X lying at -1 eV and -2 eV at ambient volume are lowered to -3 eV and -4 eV at  $v=0.50$ . Lowering of 4d character bands at K and W is also substantial. Thus Y is showing the same trends as seen in alkali metals. For Cs and related alkalis and alkaline earths under pressure, 6s character diminishes as 5d character grows strongly with pressure.<sup>33</sup> In Li, 2s character at the Fermi surface evolves to strong 2p mixture<sup>34</sup> at the volume where  $T_c$  goes above 10 K.

The Fermi surface of Y at ambient pressure (hcp) has been of interest for some time, from the pioneering calculation of Loucks<sup>28</sup> to the recent measurements and calculations of Crowe *et al.*<sup>29</sup> However, the unusual Fermi surface in the hcp structure (having a single strong nesting feature) is nothing like that in the fcc phase we are addressing, which is unusual in its own way. At  $v=1.00$  the fcc Fermi surface is a large ‘belly’ connected by wide necks along  $\langle 111 \rangle$  directions as in Cu, but in addition there are tubes (‘wormholes’) connecting a belly to a neighboring zone’s belly through each of the 24 W points. The belly encloses holes rather than electrons as in Cu; that is, the electrons are confined to a complex multiply-connected web enclosing much

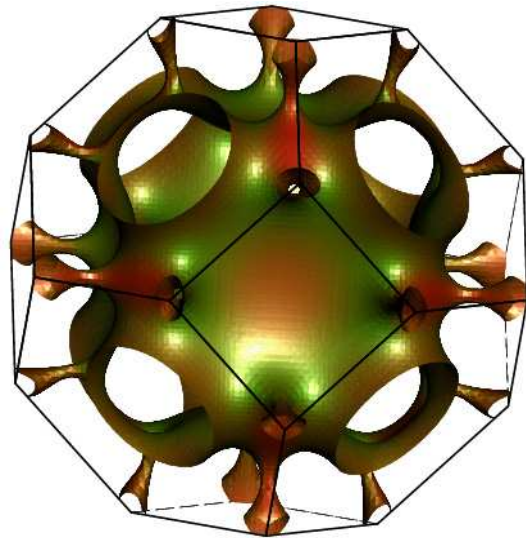


FIG. 3: (Color online) Surface plot of the Fermi surface of fcc Y at a volume corresponding to ambient pressure. The surface is shaded according to the Fermi velocity. The surface is isomorphic to that of Cu, *except* for the tubes through the W point vertices that connect Fermi surfaces in neighboring Brillouin zones. The evolution with pressure is described in the text.

of the surface of the Brillouin zone.

As the volume is reduced, the wormholes slowly grow in diameter until in the range  $0.5 < v < 0.6$ , they merge in certain places with the necks along the  $\langle 111 \rangle$  directions, and the change in topology leaves closed surfaces around the K points as well as a different complex multiply-connected sheet. The point we make is that, at all volumes, the Fermi surface is very complex geometrically. There is little hope of identifying important “nesting” wavevectors short of an extensive calculation. Even for the simple Fermi surface of fcc Li, unexpected nesting vectors were located<sup>34</sup> in three high symmetry planes of the zone. The rest of the zone in Li still remains unexplored.

#### IV. BACKGROUND: ELECTRON-PHONON COUPLING

The electron-phonon spectral function  $\alpha^2F(\omega)$  can be expressed in terms of phonon properties [and  $N(0)$ ] in the form<sup>30</sup>

$$\alpha^2F(\omega) = \frac{1}{2\pi N(0)} \sum_{\mathbf{Q}\nu} \frac{\gamma_{\mathbf{Q}\nu}}{\omega_{\mathbf{Q}\nu}} \delta(\omega - \omega_{\mathbf{Q}\nu}) \quad (1)$$

where

$$F(\omega) = \sum_{\mathbf{Q}\nu} \delta(\omega - \omega_{\mathbf{Q}\nu}) \quad (2)$$

is the density of phonon states, and  $N(0)$  is the single spin Fermi surface density of states. The phonon linewidth  $\gamma_{\mathbf{Q}\nu}$  is given by

$$\gamma_{\mathbf{Q}\nu} = 2\pi\omega_{\mathbf{Q}\nu} \sum_{\mathbf{k}} |M_{\mathbf{k}+\mathbf{Q},\mathbf{k}}^{[\nu]}|^2 \delta(\varepsilon_{\mathbf{k}}) \delta(\varepsilon_{\mathbf{k}+\mathbf{Q}}) \quad (3)$$

where  $M_{\mathbf{k}+\mathbf{Q}}^{[\nu]}$  is the electron-phonon matrix element;  $\nu$  is the branch index. Sums over  $\mathbf{Q}$  or  $\mathbf{k}$  are conventionally normalized (divided by the number of unit cells in the normalization volume).

The quantities thus defined enable one to identify the contribution to  $\lambda$  from each mode, the ‘‘mode  $\lambda$ ’’, as

$$\lambda_{\mathbf{Q}\nu} = \frac{2}{\omega_{\mathbf{Q}\nu} N(0)} \sum_{\mathbf{k}} |M_{\mathbf{k},\mathbf{k}+\mathbf{Q}}^{[\nu]}|^2 \delta(\varepsilon_{\mathbf{k}}) \delta(\varepsilon_{\mathbf{k}+\mathbf{Q}}) \quad (4)$$

With this definition  $\lambda$  is the average over the zone, and sum (not average) over the  $N_\nu = 3$  branches of all of the  $\lambda_{\mathbf{Q}\nu}$  values. The electron-phonon coupling strength  $\lambda$  then is given by

$$\lambda = \frac{4}{\pi N(0)} \sum_{\mathbf{Q}\nu} \frac{\gamma_{\mathbf{Q}\nu}}{\omega_{\mathbf{Q}\nu}^2} \equiv \sum_{\mathbf{Q}\nu} \lambda_{\mathbf{Q}\nu}. \quad (5)$$

The critical temperature  $T_c$  can be obtained by using the Allen-Dynes modification of the McMillan formula,<sup>31,32</sup> which depends on the logarithmic, first, and second frequency moments  $\omega_{log}$ ,  $\omega_1 \equiv \langle \omega \rangle$ , and  $\omega_2 \equiv \langle \omega^2 \rangle^{1/2}$ , as well as  $\lambda$  and the Coulomb pseudopotential  $\mu^*$ . These averages are weighted according to the normalized coupling ‘shape function’  $2\alpha^2 F(\omega)/(\lambda\omega)$ . They are often, and will be for Y especially under pressure, much different from simple averages over the spectrum  $F(\omega)$ .

Note that  $\lambda_{\mathbf{Q}\nu}$ , or  $\gamma_{\mathbf{Q}\nu}$ , incorporates a phase space factor, the ‘nesting function’<sup>34</sup>  $\xi(\mathbf{Q})$  describing the phase space that is available for electron-hole scattering across the Fermi surface ( $E_F=0$ ),

$$\xi(\mathbf{Q}) = \frac{1}{N} \sum_{\mathbf{k}} \delta(\varepsilon_{\mathbf{k}}) \delta(\varepsilon_{\mathbf{k}+\mathbf{Q}}) \propto \oint_{\mathcal{L}} \frac{d\mathcal{L}_{\mathbf{k}}}{|\mathbf{v}_{\mathbf{k}} \times \vec{v}_{\mathbf{k}+\mathbf{Q}}|}. \quad (6)$$

Here  $\mathcal{L}$  is the line of intersection of an undisplaced Fermi surface and one displaced by  $\mathbf{Q}$ , and  $\vec{v}_{\mathbf{k}}$  is the electron velocity at  $\vec{k}$ . These equations presume the adiabatic limit, in which the phonon frequencies are small compared to any electronic energy scale. This limit applies to elemental Y.

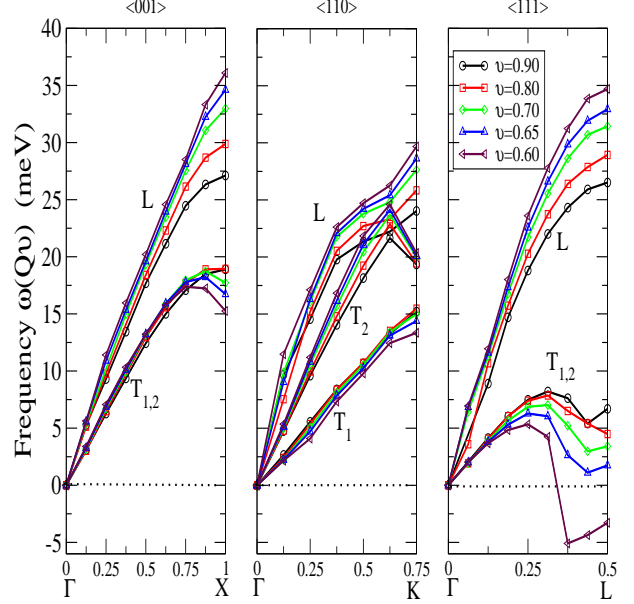


FIG. 4: (Color online) Plot of the calculated phonon spectrum along high symmetry directions ( $\Gamma$ -X,  $\Gamma$ -K,  $\Gamma$ -L) of fcc Y with different volumes. The longitudinal phonons increase with the distance from  $\Gamma$  points along all the three directions. Along  $\Gamma$ -X direction (left panel), the doubly degenerate transverse mode slightly softens near X point, while along  $\Gamma$ -K direction (left panel, only the  $T_2$  mode slightly softens near K point. Along  $\Gamma$ -L direction (right panel), the already soft doubly degenerate transverse mode soften further near the L point with decreasing volume. At  $V = 0.6V_0$ , the frequency at L becomes negative, indicating lattice instability.

## V. RESULTS AND ANALYSIS

### A. Behavior of Phonons

The calculated phonon branches are shown along the high symmetry lines, from  $v=0.90$  down to  $v=0.60$ , in Fig. 4. The longitudinal modes behave normally, increasing monotonically in frequency by  $\sim 30\%$  in this range. The transverse modes along  $\langle 100 \rangle$  and  $\langle 110 \rangle$  show little change; the doubly degenerate transverse mode at X softens by 20%, reflecting some unusual coupling. Along  $\langle 110 \rangle$ ,  $T_1$  and  $T_2$  denote modes polarized in the  $x$ - $y$  plane, and along the  $z$  axis, respectively.

The interesting behavior occurs for the (doubly degenerate) transverse branch along  $\langle 111 \rangle$ . It is quite soft already at  $v=0.9$  (7 meV, only 25% of the longitudinal branch), softer than the corresponding

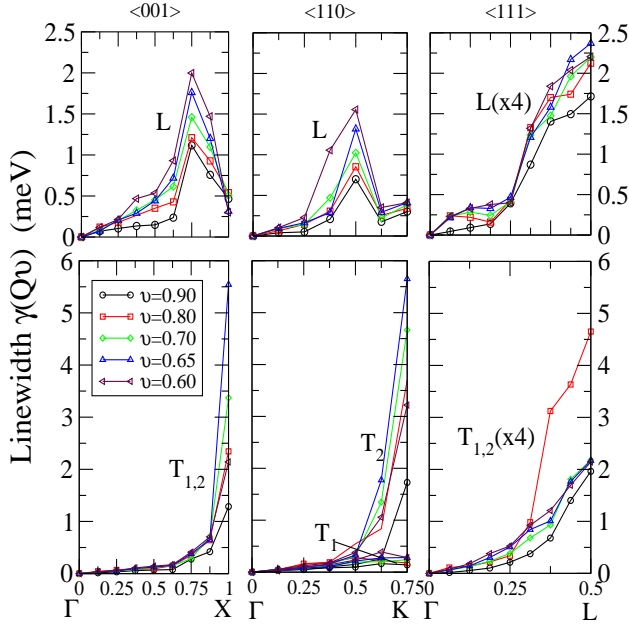


FIG. 5: (Color online) Plot of the calculated linewidths of fcc Y for varying volumes. The linewidths of the transverse modes at the X point increases from 1.3 to 5.5 as volume decreases from  $V=0.9V_0$  to  $V=0.6V_0$ . The linewidths of the  $T_2$  along  $\langle 110 \rangle$  modes show the same increase. The linewidths along the  $\langle 111 \rangle$  direction have been multiplied by four for clarity.

mode in hcp Y at ambient pressure.<sup>35</sup> With decreasing volume it *softens* monotonically, and becomes unstable between  $v=0.65$  and  $v=0.60$ . It should not be surprising that the transverse mode at the L point is soft in a rare earth metal. The sequence of structural transitions noted in the introduction (typically hcp $\rightarrow$ Sm-type $\rightarrow$ dhcp $\rightarrow$ dfcc $\rightarrow$ fcc for trivalent elements) involves only different stacking of hexagonal layers of atoms along the cubic (111) direction. So although these various periodic stackings may have similar energies, the soft (becoming unstable) transverse mode at L indicates also that the barrier against sliding of these planes of atoms is very small. At  $v=0.60$  (see Fig. 4) the largest instability is not at L itself but one-quarter of the distance back toward  $\Gamma$ . At  $v=0.65$  there are surely already anharmonic corrections to the lattice dynamics and coupling from the short wavelength transverse branches.

## B. Linewidths

The linewidths  $\gamma_{Q\nu}$ , one indicator of the mode-specific contribution to  $T_c$ , are shown in Fig. 5. To understand renormalization, one should recognize that in lattice dynamical theory it is  $\omega^2$ , and not  $\omega$  itself, that arises naturally. At  $v=0.90$ ,  $\omega^2$  for the T modes is only 1/16 of the value for the longitudinal mode at the L point. A given amount of coupling will affect the soft modes much more strongly than it does the hard modes.

For the  $\langle 110 \rangle$  direction, the strong peak in  $\gamma_{Q\nu}$  for the  $T_2$  ( $\hat{z}$  polarized) mode at the zone boundary point K (5.7 meV) is reflected in the dip in this mode at K that can be seen in Fig. 4. At  $v=0.60$  the linewidth is 1/3 of the frequency. The coupling to the  $T_1$  mode along this line is negligible. Note that it is the  $T_1$  mode that is strongly coupled in Li and is the first phonon to become unstable. A peak in the linewidth of the L modes correlates with a depression of the frequency along this line. Along  $\langle 001 \rangle$  the T modes again acquire large linewidths near the zone boundary under pressure. This electron-phonon coupling is correlated with the dip in the T frequency in the same region.

The coupling along the  $\langle 111 \rangle$  direction is not so large, either for T or for L branches (note, they have been multiplied by a factor of four in Fig. 5). The coupling is strongest at the zone boundary, and coupled with the softness already at  $v=0.90$ , the additional coupling causes an instability when the volume is reduced to  $v=0.60$  ( $P = 42$  GPa). This seems to represent an example where a rather modest amount of coupling has a potentially catastrophic result: instability of the crystal. Evidently Y is stabilized in the fcc structure by anharmonic effects, coupled with the fact that being already close-packed there may be no simple structural phase that is lower in energy.

## C. Coupling Strength

It is intuitively clear that strong coupling to extremely low frequency modes is not as productive in producing high  $T_c$  as coupling to higher frequency modes. This relationship was clarified by Bergmann and Rainer,<sup>36</sup> who calculated the functional derivative  $\delta T_c / \delta \alpha^2 F(\omega)$ . They found that coupling at frequencies less than  $\bar{\omega} = 2\pi T_c$  has little impact on  $T_c$  (although coupling is never harmful). Since we are thinking in terms of Y's maximum observed  $T_c \approx 20$  K, this means that coupling below  $\bar{\omega} = 10$  meV

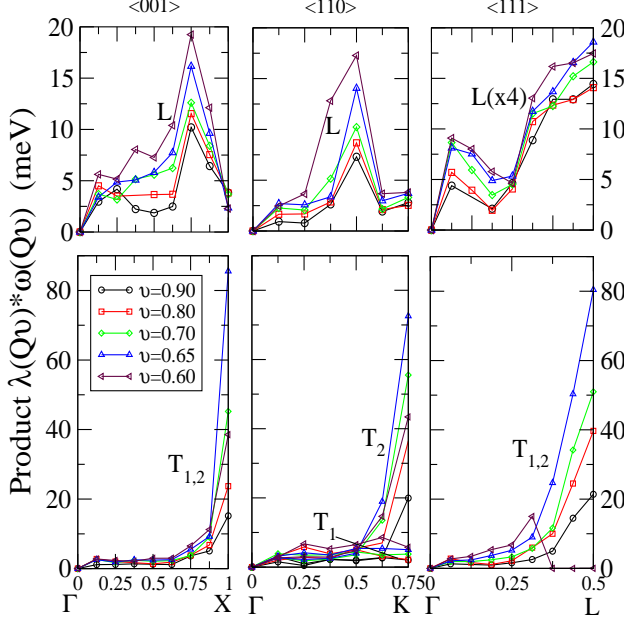


FIG. 6: (Color online) Plot of the product  $\lambda_{Q\nu}\omega_{Q\nu}$  of fcc Y for different volumes, along the high symmetry directions. Note that the longitudinal (L) values along  $\langle 111 \rangle$  have been multiplied by four for clarity. In addition, values corresponding to unstable modes near L have been set to zero. Differences in this product reflect differences in matrix elements; see text.

becomes ineffective.

The product  $\lambda_{Q\nu}\omega_{Q\nu} \propto \gamma_{Q\nu}/\omega_{Q\nu}$  gives a somewhat different indication of the relative coupling strength<sup>37</sup> than does either  $\lambda_{Q\nu}$  or  $\gamma_{Q\nu}$ . It is also, up to an overall constant, just the nesting function defined earlier, with electron-phonon matrix elements included within the sum. Since the nesting function is a reflection of the phase space for scattering, it is independent of the polarization of the mode, hence differences between the three branches are due solely to the matrix elements.

This product  $\lambda_{Q\nu}\omega_{Q\nu}$  is shown in Fig. 6. The weight in the transverse modes is concentrated near the zone boundary, with the region being broader around L than at X or K and growing in width with pressure. The  $T_1$  branch along  $\langle 110 \rangle$ , which is polarized along  $[1\bar{1}0]$ , shows essentially no coupling. The weight in this product for the longitudinal modes is peaked *inside* the zone boundary along the  $\langle 001 \rangle$  and  $\langle 110 \rangle$  directions with a mean value of 7-8 meV, and is weaker along the  $\langle 111 \rangle$  direction.

TABLE I: For each volume  $v$  studied, the columns give the experimental pressure<sup>22</sup>(GPa), the Fermi level density of states  $N(0)$  (states/Ry spin), and calculated values of the mean frequency  $\omega_1 = \langle \omega \rangle$  (meV), the logarithmic moment  $\omega_{log}$  and second moment  $\omega_2 = \langle \omega^2 \rangle^{1/2}$  (all in meV), the value of  $\lambda$ , the product  $\lambda\omega_2^2$  (meV<sup>2</sup>), and  $T_c$  (K). For  $T_c$  the value of the Coulomb pseudopotential was taken as  $\mu^*=0.15$ .

v	P	N(0)	$\omega_{log}$	$\omega_1$	$\omega_2$	$\lambda$	$\lambda\omega_2^2$	$T_c$
0.90	6	9.7	12.5	13.6	14.7	0.75	162	4.0
0.80	14	11.3	11.6	13.2	14.5	1.30	273	11.9
0.70	26	9.1	10.1	12.0	13.8	1.53	291	13.0
0.65	32	8.4	7.6	10.2	12.6	2.15	341	14.4
0.60	42	7.9	6.9	9.5	12.1	2.80	410	(16.9)

#### D. $\alpha^2 F(\omega)$

The results for  $\alpha^2 F$  are displayed in Fig. 7. The longitudinal peak in the 20-35 range hardens normally with little change in coupling strength. The 7-20 meV range of transverse modes at  $v=0.90$  increases in width to 2-24 meV at  $v=0.60$ , and the strength increases monotonically and strongly. The strong peak in  $\alpha^2(\omega) = \alpha^2 F(\omega)/F(\omega)$ , shown in the bottom panel of Fig. 7, reflects the very soft modes that have been driven into the 2-5 meV range, and the fact that they are very relatively strongly coupled. The substantial increase in coupling, by a factor of  $\sim 2.5$ , in the range 7-25 meV is important for  $T_c$ , as noted in the next subsection.

#### E. Estimates of $T_c$

This background helps in understanding the trends displayed in Table I, where  $T_c$  from the Allen-Dynes equation<sup>31</sup> (choosing the standard value of  $\mu^*=0.15$ ) and the contributing materials constants are displayed. The calculated values of  $\lambda$  increases strongly, by a factor of 3.7 in the volume range we have studied. Between  $v=0.65$  and  $v=0.60$  (the unstable modes are removed from consideration)  $\lambda$  increases 30% but  $T_c$  increases by only 2.5 degrees. The cause becomes clear in looking at the frequency moments. These moments are weighted by  $\alpha^2 F(\omega)/\omega$ .  $\alpha^2(\omega)$  itself become strongly peaked at low frequency under pressure, and it is further weighted by  $\omega^{-1}$ . The frequency moments set the scale for  $T_c$  ( $\omega_{log}$  in particular) and they *decrease* strongly with decreasing volume. In particular,  $\omega_{log}$  decreases by 45% over the volume range we have studied, reflecting its strong sensitivity to soft

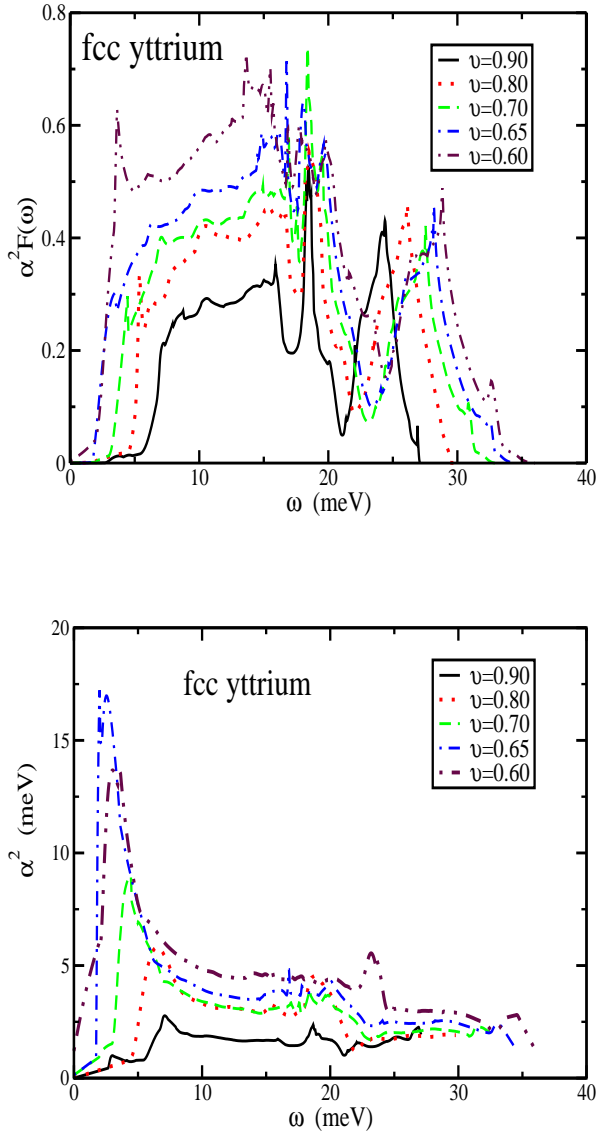


FIG. 7: (Color online) Top panel: Plot of  $\alpha^2 F(\omega)$  versus  $\omega$ . As volume decreases,  $\alpha^2 F(\omega)$  increases and gradually transfers to low frequency. Bottom panel: the frequency-resolved coupling strength  $\alpha^2(\omega)$  for each of the volumes studied. The evolution with increased pressure is dominated by strongly enhanced coupling at very low frequency (2-5 meV).

modes.

The increase in  $T_c$  probably owes more to the increase in coupling in the 10-25 meV range (see  $\alpha^2(\omega)$  plot in Fig. 7; a factor of roughly 2.5) than to the more spectacular looking peak at very low frequency. Put another way, the very low frequency peak in  $\alpha^2 F$  looks impressive and certainly contributes strongly

to  $\lambda$ , but is also very effective in lowering the temperature scale ( $\omega_{log}$ ). For  $\alpha^2(\omega)$  shapes such as we find for Y, the quantities  $\lambda$  and  $\omega_{log}$  which go into the Allen-Dynes equation for  $T_c$  do not provide a very physical picture of the change in  $T_c$ . For this reason we provide also in Table I the product  $\lambda\omega_2^2 = N(0) \langle I^2 \rangle / M$  ( $\langle I^2 \rangle$  is the conventional Fermi surface average of square of the electron-ion matrix element and  $M$  is the atomic mass). For the volumes  $0.60 \leq v \leq 0.80$  in the table, the ratio of  $\lambda\omega_2^2/T_c$  is nearly constant at  $23 \pm 1.5$  (in the units of the table), illustrating the strong cancellation of the increase of  $\lambda$  with the decrease in frequency moments in producing the resulting  $T_c$ .

## VI. SUMMARY

In this paper we have presented the evolution of elemental Y over a range of volumes ranging from low pressure to 40+ GPa pressure ( $V/V_o = 0.60$ ). Lattice instabilities that emerge near this pressure (and persist to higher pressures) make calculations for smaller volumes/higher pressures unrealistic. For simplicity in observing trends the structure has been kept cubic (fcc); however, the observed phases are also close-packed so it was expected that this restriction may still allow us to obtain the fundamental behavior underlying the unexpectedly high  $T_c$  in Y. On the other hand, the Fermi surface geometry varies strongly with crystal structure, and the nesting function  $\xi(Q)$  and perhaps also the matrix elements may have some sensitivity to the type of long-range periodicity.

In addition to the band structure, Fermi surface, and electronic density of states, we have also presented the phonon dispersion curves and linewidths along the high symmetry directions, and also have presented  $\alpha^2 F(\omega)$  and the resulting value of  $T_c$ . The results show indeed that Y under pressure becomes a strongly coupled electron-phonon system, readily accounting for value of  $T_c$  in the range that is observed.

In spite of having used a relatively dense mesh of  $Q$  points for the phonons, it seems clear that this Brillouin zone integral is still not well converged. Evaluation of  $\xi(Q)$  on a very fine  $Q$  mesh in three planes for fcc Li, which has a very simple Fermi surface, has shown<sup>34</sup> that this nesting factor contains (thickened) surfaces of fine structure with high intensity. The convergence of this zone integral (and for example the resulting  $\alpha^2 F$  function) has rarely been tested carefully in full linear response evaluations of

phonons; such a test could be very computationally intensive. Nevertheless, the general finding of strong coupling is clear.

Very recently it has been found that isovalent Sc is superconducting at 8.1 K under 74 GPa pressure.<sup>38</sup> Note that if the lattice were harmonic and the only difference between Sc and Y were the masses (which differ by a factor of two),  $T_c = 20$  K for Y would translate to  $T_c = 28$  K for Sc. (For an element with a harmonic lattice,  $\lambda$  is independent of mass.) The corresponding argument for (again isovalent) La gives  $T_c = 16$  K. La has  $T_c = 13$  K at 15 GPa, and has not been studied beyond<sup>7</sup> 45 GPa.

Another comparison may be instructive. Dynes and Rowell obtained and analyzed tunneling data<sup>42</sup> on Pb-Bi alloys where  $\lambda$  is well into the strong coupling region, becoming larger than two as is the case for Y under pressure in Table I. The  $\text{Pb}_{0.65}\text{Bi}_{0.35}$  alloy has  $\lambda=2.13$ ,  $\langle \omega^2 \rangle = 22.6$  meV<sup>2</sup>. We can compare directly with the  $v=0.65$  case in Table I, which has  $\lambda=2.15$ ,  $\langle \omega^2 \rangle = 159$  meV<sup>2</sup>. The product  $M \langle \omega^2 \rangle$  for Y is three times as large as for the heavy alloy. Since the  $\lambda$ 's are equal, the value of  $N(0) \langle I^2 \rangle$  (equal to  $\lambda M \langle \omega^2 \rangle$ ) is also three times as large as in the alloy. The values of  $T_c$  are 14.4 K (Y) and 9 K (alloy) [somewhat different values of  $\mu^*$  were used.] The values of  $\omega_{log}$  differ by less than a factor of two, due to the low-frequency coupling in  $\alpha^2(\omega)$  in Y that brings that frequency down, and that is why the values of  $T_c$  also differ by less than a factor of two.

While this study is in some sense a success, in that it has become clear that strong electron-phonon coupling can account for the remarkable superconductivity of Y under pressure, there remains a serious shortcoming, one that is beyond the simple lack of numerical convergence that would pin down precisely  $\lambda$ ,  $T_c$ , etc. What is lacking is even a rudimentary physical picture for what distinguishes Y and Li ( $T_c$  around 20 K under pressure) from other elemental metals which show low, or vanishingly small, values of  $T_c$ .

The rigid muffin-tin approximation (RMTA) of Gaspari and Gyorffy,<sup>39</sup> which approximates the phonon-induced change in potential and uses an isotropic idealization for the band structure to derive a simple result, seemed fairly realistic for the electronic contribution (the Hopfield  $\eta$ ) for transition metal elements and intermetallics.<sup>40,41</sup> On top of these idealizations, there is an additional uncertainty in  $\langle \omega^2 \rangle$  that must be guessed to obtain  $\lambda$  and  $T_c$ . One would not 'guess' the values of the frequency moments that we have obtained for Y under pressure.

In addition, the RMTA expression does not distinguish between the very different matrix elements for the various branches, giving only a polarization and Fermi surface average. Nevertheless, it gave a very useful understanding of trends<sup>40</sup> in electron-phonon coupling in elemental transition metals and in some intermetallic compounds. While the linear response evaluation of the phonon spectrum and the resulting coupling seems to work well, this more detailed approach has not yet provided – even for elemental superconductors – the physical picture and simple trends that would enable us to claim that we have a clear understanding of strong coupling superconductivity.

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