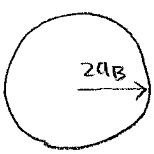
PHYS 140B HW # 4

1.

	S	L	J	Symbol	$g\sqrt{s(s+1)}$	$g\sqrt{J(J+1)}$
$2p^0$	0	0	0	$^1S_0$	0	0
$2p^1$	$\frac{1}{2}$	1	$\frac{1}{2}$	$^2P_{1/2}$	0.58	0.58
$2p^2$	1	1	0	$^3P_0$	2.83	0
$2p^3$	$\frac{3}{2}$	0	$\frac{3}{2}$	$^4S_{3/2}$	3.87	3.87
$2p^4$	1	1	2	$^3P_2$	2.12	3.67
$2p^5$	$\frac{1}{2}$	1	$\frac{3}{2}$	$^2P_{3/2}$	1.15	2.58
$2p^6$	0	0	0	$^1S_0$	0	0

2.



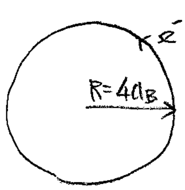
$$\Delta x = \left[\frac{1}{2\pi}\int_0^{2\pi} x^2\, d\theta\right]^{1/2} = \left[\frac{R^2}{2}\right]^{1/2} = \frac{2a_B}{\sqrt{2}} = \sqrt{2}\, a_B$$ (factors of 2, $\sqrt{3}$.... are not important)

Heisenberg uncertainty relation : $\Delta x \Delta P_x \sim \hbar$, so we have $\Delta P_x \sim \frac{\hbar}{\Delta x} = \frac{\hbar}{\sqrt{2}a_B}$

$$\Delta P_x = \sqrt{<(P_x - <P_x>)^2>} = \sqrt{<P_x^2> - <P_x>^2} = \sqrt{<P_x^2>}$$ because $<P_x>$ vanishes.

Then we can calculate the characteristic velocity as :

$$\Delta P_x = \sqrt{(m v_c)^2} \implies v_c = \frac{\Delta P_x}{m} = \frac{\hbar}{\sqrt{2}\, m a_B}$$



current $I = \frac{dQ}{dt} = \frac{e}{T} = \frac{e v_c}{2\pi R} = \frac{e\hbar}{2\pi(4a_B)\cdot\sqrt{2}\, m a_B}$

and magnetic moment $M = I \times \text{Area}$

$$= I \times \pi R^2$$

$$M = \frac{e\hbar}{(2\pi)(4a_B)\sqrt{2}\, m a_B} \cdot \pi \cdot (4a_B)^2 = \frac{4e\hbar}{2\sqrt{2}\, m} = \frac{4}{\sqrt{2}}\mu_B = 2\sqrt{2}\,\mu_B$$

The magnetic moment is of the order of magnitude of μ_B !

3. $M = M_{sat} B_J \left(\dfrac{g \mu_B J B}{k_B T} \right)$ eqn. (9.23)

$$B_J(x) = \frac{2J+1}{2J} \coth \frac{(2J+1)x}{2J} - \frac{1}{2J} \coth \frac{x}{2J}$$

As $J \to \infty$ $\quad \dfrac{2J+1}{2J} \to 1, \quad \dfrac{(2J+1)x}{2J} \to x$

$$\frac{1}{2J} \coth \frac{x}{2J} \to \frac{e^{\frac{x}{2J}} + e^{-\frac{x}{2J}}}{(2J)\left(e^{\frac{x}{2J}} - e^{-\frac{x}{2J}}\right)}$$

$$\to \frac{1 + \frac{x}{2J} + \cdots \quad 1 - \frac{x}{2J} + \cdots}{2J\left(1 + \frac{x}{2J} + \cdots - 1 + \frac{x}{2J} - \cdots\right)}$$

$$\to \frac{2}{2x} = \frac{1}{x}$$

$\therefore$ $B_{J\to\infty}(x) = \coth x - \dfrac{1}{x}$, this is the Langevin function $L(x)$

$$M(J\to\infty) = M_{sat} \left\{ \coth\left(\frac{g\mu_B J B}{k_B T} \right) - \frac{k_B T}{g \mu_B J B} \right]$$

$$= M_{sat}\, L\left(\frac{g \mu_B J B}{k_B T} \right)$$

4. eqn. (9.59) $\quad \chi_p = \mu_0 \mu_B^2 \rho(E_F)$ $\quad \rho(E_F) = \dfrac{3n}{2E_F}$

$$= \mu_0 \mu_B^2 \frac{3n}{2E_F} \qquad E_F = k_B T_F$$

$$= \mu_0 \mu_B^2 \frac{3n}{2} \cdot \frac{1}{k_B T_F}$$

$$= \frac{C}{T_F}$$

where $C \equiv \dfrac{3n \mu_0 \mu_B^2}{2 k_B}$