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On heavy carbon doping of MgB₂

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7 Abstract

8 Heavy carbon doping of MgB₂ is studied by first principles electronic structure studies of two types, an ordered
 9 supercell (Mg(B_{1-x}C_x)₂, $x = 0.0833$) and also the coherent potential approximation method that incorporates effects
 10 of B–C disorder. For the ordered model, the twofold degenerate σ -bands that are the basis of the high temperature
 11 superconductivity are split by 60 meV (i.e. 7 meV/% C) and the σ Fermi cylinders contain 0.070 holes/cell, compared
 12 to 0.11 for MgB₂. A virtual crystal treatment tends to overestimate the rate at which σ holes are filled by substitutional
 13 carbon. The coherent potential approximation (CPA) calculations give the same rate of band filling as the supercell
 14 method. The occupied local density of states of C is almost identical to that of B in the upper 2 eV of the valence bands,
 15 but in the range -8 eV to -2 eV, C has a considerably larger density of states. The calculations indicate that the σ
 16 Fermi surface cylinders pinch off at the zone center only above the maximum C concentration $x \approx 0.10$. These results
 17 indicate that Mg(B_{1-x}C_x)₂ as well as Mg_{1-x}Al_xB₂ is a good system in which to study the evolution of the unusual elec-
 18 tron–phonon coupling character and strength as the crucial σ hole states are filled.
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21 1. Introduction

22 Discovery of superconductivity at $T_c \approx 40$ K in
 23 MgB₂ has enlivened not only interest in new classes
 24 of superconductors with high T_c and novel two-gap
 25 behavior, but also pursuit of new materials for
 26 applications that require high critical current densi-
 27 ties and high critical fields (H_{c2}). In highly resistive

films (due to unreacted components and/or oxygen 28
 and carbon impurities) the perpendicular and par- 29
 allel critical fields have reached [1] $H_{c2}^{\perp} \approx 34$ T, 30
 $H_{c2}^{\parallel} \approx 49$ T, making MgB₂ a real possibility for a 31
 high field conductor. Such application require de- 32
 fects or grain boundaries to pin the vortices whose 33
 motion would otherwise lead to energy dissipation 34
 and joule heating. Such defects also affect the 35
 underlying electronic structure and pairing interac- 36
 tion, at least in some parts of the sample, and often 37
 decrease T_c . The simplest defect to understand is 38
 the substitutional impurity, which provides a 39

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40 means of varying the intrinsic properties in a con-
41 tinuous and controllable manner and may enhance
42 pinning mechanisms. Although readily synthesiz-
43 able as a stoichiometric, rather clean compound,
44 the MgB_2 lattice resists most attempts to alloy,
45 both on the Mg and B sites, at more than the very
46 dilute level.

47 The two established exceptions are Al substitu-
48 tion for Mg, and C substitution for B, both of
49 which lead to a rapid decrease in T_c . Published re-
50 ports on the behavior vary considerably with syn-
51 thesis method and sample treatment, but some of
52 the general behavior seems to be established. Al
53 substitution should be the simpler one, since the
54 very strongly bonded honeycomb structure B lay-
55 ers remain intact, and simple rigid band filling of
56 the hole states at first appears to be a likely possi-
57 bility. Up to $y = 0.10$ in $\text{Mg}_{1-y}\text{Al}_y\text{B}_2$ that appears
58 to be the case, with T_c decreasing smoothly. Be-
59 yond the concentration $y = 0.10$, however, there
60 are signs of two-phase behavior (two c lattice
61 parameters in diffraction studies) arise [2]. Around
62 $y = 0.25$ the system reverts to a single phase very
63 low and finally vanishing T_c . A study of the trends
64 in electron–phonon coupling strength using first
65 principles calculations in the virtual crystal
66 approximation [3] reported a sharp change of
67 behavior at $y = 0.25$. A thorough study of the
68 energetics of Al substitution revealed a strong ten-
69 dency for superstructure formation [4] at $y = 0.25$
70 (and $y = 0.75$). Still, the observed onset of two-
71 phase behavior already at $y = 0.10$ is unexplained,
72 and nonstoichiometry [5] and microscopic defects
73 must be kept in mind.

74 Unlike the Al alloying case where there has
75 been at least rough consensus on changes of prop-
76 erties, reports of the change in T_c and structure
77 with addition of carbon on MgB_2 have varied
78 widely. These differences seem in some cases to re-
79 flect real differences in materials due to the various
80 methods of synthesis and heat treatment, and the
81 samples prepared were highly mixed phase, there-
82 by creating additional uncertainty. The very simi-
83 lar X-ray scattering strength of the B and C
84 atoms has rendered standard X-ray diffraction
85 ineffective in determining the C content of a sam-
86 ple. Reports early on suggested C miscibilities in
87 $\text{Mg}(\text{B}_{1-x}\text{C}_x)_2$ as small [6] as $x = 0.02$ to as large

[7] as $x = 0.3$. Recent studies of several groups sug- 88
gest that $x = 0.2$ can be achieved while larger con- 89
centrations are questionable [8–10] although 90
reports of larger C concentrations under high pres- 91
sure growth persist [11]. 92

93 With some of the materials questions coming
94 under control, the observation trends are raising
95 some serious questions. The decrease in T_c follows
96 $dT_c/dx \sim 1 \text{ K}/\% \text{ C}$ [12], with several reports of
97 $T_c \approx 21\text{--}23 \text{ K}$ at $x = 0.2$ [8–10]. However, with car-
98 bon adding one electron and the σ -bands of MgB_2
99 holding only ~ 0.11 holes per unit cell [13], one
100 might guess that the σ -bands would be filled (or
101 nearly so, as the π -bands can also accept carriers)
102 and superconductivity would have vanished. Not
103 only is T_c still robust at $x = 0.2$, analysis of tunnel-
104 ing spectra indicates two-band superconductivity
105 is retained [14,15], whereas it might also seem
106 likely that disorder scattering should have aver-
107 aged out the gap to a single value. The upper crit-
108 ical field H_{c2} initially increases strongly with C
109 content as T_c is depressed, reaching a maximum
110 [1,12,16] of $\sim 33\text{--}35 \text{ T}$. This critical field is substan-
111 tially less than that for more disordered films (see
112 above).

113 Recent studies seem to agree that C can be
114 introduced substitutionally for B up to 10%
115 replacement $x = 0.10$ [8,17]. Tunneling spectroscopies
116 indicate that $\text{Mg}(\text{B}_{1-x}\text{C}_x)_2$ remains a two-gap
117 superconductor [8,14] (i.e. the gap anisotropy is
118 not washed out by scattering) for $10 \pm 2\% \text{ C}$ sub-
119 stitution for B. This question of how the anisot-
120 ropy gets washed out has attracted much
121 interest. Similarly, the mechanisms underlying
122 the high critical fields are not understood. These
123 questions are not the focus of the present study,
124 although the knowledge gained from first princi-
125 ples calculations will be useful in the resolution
126 of these topics.

127 A few first principles calculations have been
128 done for C substitution of B in MgB_2 . Pseudopo-
129 tential calculations including relaxation in a 27
130 unit cell supercell [18] confirmed that substitu-
131 tional C is energetically favorable to interstitial
132 C, and that the C–B bond length is 5–6% shorter
133 than the bulk B–B bond length. A projector-aug-
134 mented-wave calculation [19] reported a smaller
135 relaxation, but the difference may be due to con-

136 strains related to supercell size. A coherent poten-
 137 tial approximation study of the disordered alloy
 138 using a Korringa–Kohn–Rostoker multiple scat-
 139 tering method in the atomic sphere approximation
 140 [20] revealed relatively small effects of disorder in
 141 the range $0 \leq x \leq 0.3$. The main effect of C substi-
 142 tution was reported to be the raising of the Fermi
 143 energy due to the additional carriers, but the filling
 144 of the crucial σ hole band was not quantified.

145 In this paper, we present a more detailed analy-
 146 sis of the changes in electronic structure, and the
 147 effective σ -band doping, using both periodic
 148 supercells and disordered alloy calculations. Our
 149 study lays the groundwork for another issue in
 150 the superconductivity of MgB_2 alloys that has re-
 151 ceived no direct attention from experimentalists:
 152 the effect of decreasing holes on the strength and
 153 character of electron–phonon coupling. Theoretic-
 154 al studies [21–23] have predicted the following
 155 remarkable changes in doped MgB_2 : as the num-
 156 ber of holes decreases (hence the Fermi wavevector
 157 k_F of the cylindrical σ Fermi surfaces decreases),
 158 (1) the substantial downward renormalization of
 159 the E_{2g} bond-stretching modes with $Q \leq 2k_F$ does
 160 not change, (2) the coupling strengths of these al-
 161 ready strongly coupled modes increases, and (3)
 162 the total coupling strength λ , and hence T_c , re-
 163 mains unchanged. This scenario assumes two-
 164 dimensionality of the σ -bands which is only
 165 approximately true, and neglects the change of
 166 electron–phonon matrix elements which should
 167 be reasonable. This argument also neglects the fact
 168 that the applicability of conventional electron–
 169 phonon (Midgal–Eliashberg) theory, already
 170 somewhat suspect [24,25] in MgB_2 , definitely be-
 171 comes inapplicable as the Fermi energy decreases
 172 and approaches more closely the bond-stretching
 173 mode energy (65 meV). Item (2) definitely portends
 174 unusual dynamics related to the B–B stretch
 175 modes. We hope the current paper stimulates
 176 experimental investigation into these questions.

177 2. Computational methods

178 Two methods of assessing the effects of substi-
 179 tutional C in MgB_2 have been used in the work re-
 180 ported here. Our ordered impurity calculations

Table 1

Lattice parameter values used in the virtual crystal and supercell calculations

Lattice parameter	Undoped MgB_2 (Å)	10% Doping $\text{MgB}_{1.8}\text{C}_{0.2}$ [17] (Å)	8.33% Doping $\text{Mg}_6\text{B}_{11}\text{C}$ (Å)
a	3.083	3.053	$3 \times 3.053 = 49.159$
b	3.083	3.053	$2 \times 3.053 = 6.106$
c	3.521	3.525	3.525

181 have been performed using the full-potential line-
 182 arized augmented plane wave code Wien2k [26],
 183 applying the generalized gradient approximation
 184 [27] to the exchange–correlation potential. The ba-
 185 sis set is reduced by using the APW + lo method
 186 [28], retaining the accuracy of the LAPW method.
 187 RK_{max} was set to 7.00 which is a high quality basis
 188 for the s–p electron systems. The Brillouin zone
 189 integration for the supercell was carried out using
 190 432 k points in the irreducible part of the zone. We
 191 have used the experimental results of Avdeev et al.
 192 [17] for the lattice parameters of the 8.33% doped
 193 supercell and the 10% doped virtual crystal (Table
 194 1).

195 For the disordered (randomly substituted) alloy
 196 calculations, we have used the full-potential non-
 197 orthogonal local-orbital minimum-basis scheme
 198 (FPLO) [29] applied in the coherent potential
 199 approximation (CPA) [30]. In the calculations,
 200 1152 ($45 \times 45 \times 10$) irreducible k points, and va-
 201 lence orbitals 2s2p3s3p3d for Mg and 2s2p3d for
 202 B and C, were used. The implementation of the
 203 CPA in FPLO relies on the Blackman–Esterling–
 204 Berk theory [31] that includes random off-diagonal
 205 matrix elements in the local-orbital representation.

206 3. Ordered $\text{Mg}(\text{B}_{1-x}\text{C}_x)_2$, $x = 0.0833$

207 Smaller concentrations of ordered C substitui-
 208 tion for B require larger supercells. Our choice of
 209 supercell was guided by the desire for a small en-
 210 ough C concentration to be relevant to address
 211 experimental data, yet not so small as to make it
 212 difficult to distinguish the effect of C addition or
 213 to make the calculations unreasonably tedious.
 214 Since interlayer hopping is not an issue we will ad-
 215 dress, the supercell will involve enlargement of

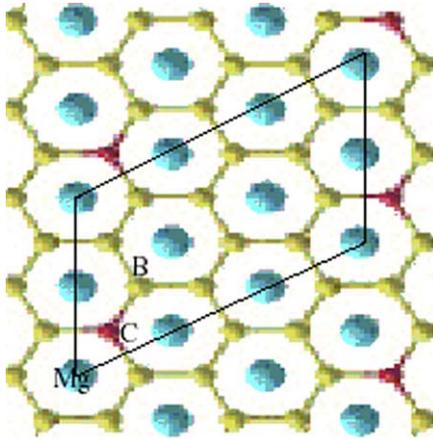


Fig. 1. Top view of the B–C layer of the $\text{Mg}_6\text{B}_{11}\text{C}$ supercell. Large (blue) circles denote Mg atoms, small light (yellow) circles denote boron, and small dark (red) circles denote carbon. The supercell boundary is outlined. (For interpretation of the references in colour in this figure legend, the reader is referred to the web version of this article.)

216 only a single B layer. The shape of supercell
 217 should have as small as possible aspect ratio in
 218 order to maximize the separation of C atoms.
 219 The compromise we chose was the 8.33% doped
 220 supercell ($\text{Mg}_6\text{B}_{11}\text{C} \rightarrow \text{Mg}(\text{B}_{1-x}\text{C}_x)_2$, $x = 1/12 =$
 221 0.0833). Fig. 1 shows the top view of the B_{11}C
 222 layer in the 2×3 supercell. Each carbon atom
 223 has three first, five second, and three third nearest
 224 neighbor boron atoms.

225 3.1. Band structure

226 There are two effects of carbon substitution: a
 227 change in the average potential in the B–C layer,
 228 and the breaking of symmetry by C replacement
 229 of B in the supercell. In the ordered supercell the
 230 potential difference is kept explicit and symmetry
 231 breaking will be clear. In Section 6 CPA will be
 232 used to probe these effects in a different manner.
 233 Another effect, B relaxation around the C impuri-
 234 ties, has been addressed previously to some extent,
 235 but it will not be studied here since the effect can-
 236 not be included within the CPA.

237 The band structure within 4 eV of the Fermi le-
 238 vel (E_F) is shown in Fig. 2. The primary band in
 239 the understanding of MgB_2 superconductivity is

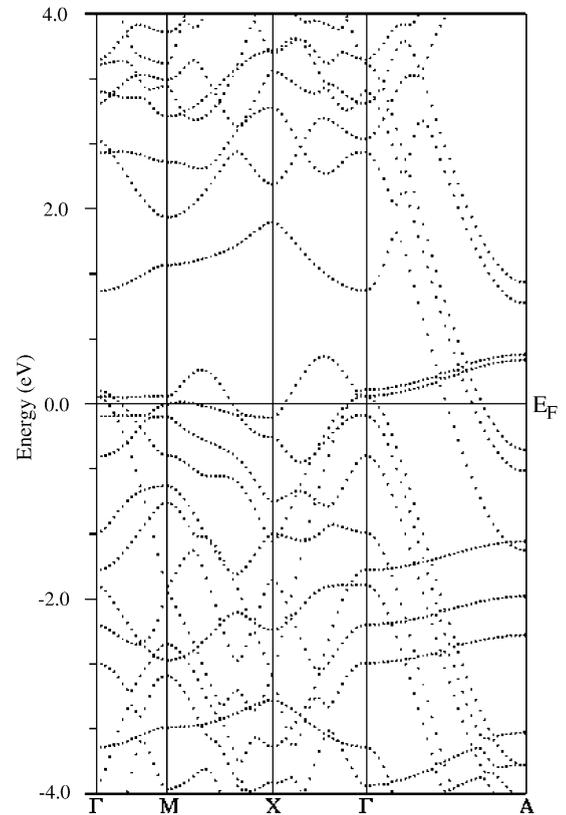


Fig. 2. Band structure of $\text{Mg}_6\text{B}_{11}\text{C}$ in the region of the Fermi level. The twofold degenerate σ -band in the undoped system along the Γ – A is split by 60 meV when 1/12 of B is replaced by C.

the (twofold degenerate) σ -band along the Γ – A 240
 direction. The results of supercell band-folding 241
 can be distinguished especially for the Γ – A direc- 242
 tion, where (for example) the σ -band is replicated 243
 at Γ at -1.8 eV, -2.3 eV, -2.6 eV, and even fur- 244
 ther lower energies. 245

One of the most readily apparent effects of C 246
 doping is the splitting of the σ -bands along Γ – A , 247
 by 60 meV. It is helpful in understanding this split- 248
 ting to obtain the atomic characters of each of the 249
 split bands, which will in any case involve only the 250
 B and C $p\sigma$ (p_x, p_y) states. The carbon p contribu- 251
 tion is emphasized by enlarged symbols (“fat- 252
 bands”) in Fig. 3. The lower of the two bands 253
 has stronger C content (although this is not easy 254
 to distinguish in Fig. 3). In addition, it has more 255
 first and second B neighbor character than does 256

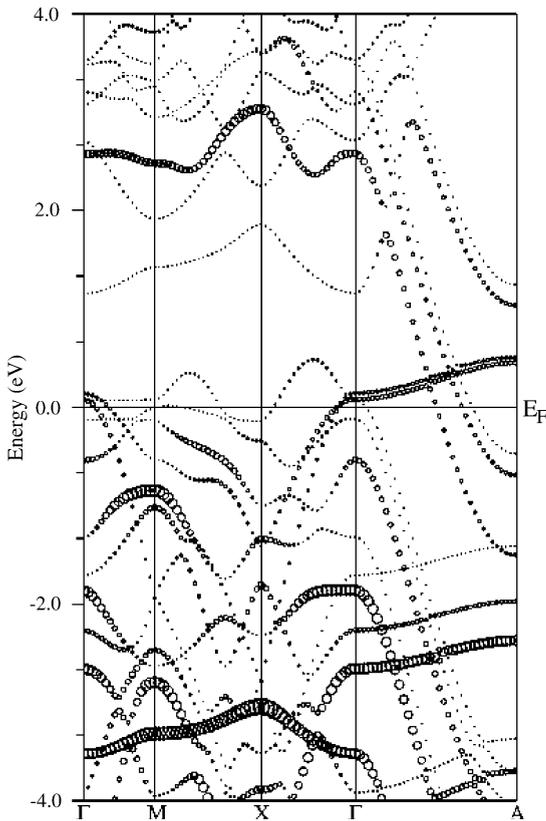


Fig. 3. Bands of $Mg_6B_{11}C$ near the Fermi level, with symbol size proportional to the C 2p character. The σ -bands, split by 60 meV (hardly visible in this figure) lie just above E_F at Γ and disperse upward toward the A point.

257 the upper band. The upper band contains more
 258 third neighbor B character, this being the B site
 259 farthest from the C atom and representative of
 260 the bulk material. The interpretation of this split-
 261 ting is that lower band has been pulled down due
 262 to the stronger potential of the C atom compared
 263 to that of B. The 60 meV splitting for $x = 1/12$ pro-
 264 vides (assuming a linear effect in this concentration
 265 range) an energy scale for σ -band broadening
 266 $\gamma_0 \approx 7$ meV/% C content.

267 3.2. Density of states

268 Fig. 4 shows the total and atom-projected den-
 269 sity of states (DOS) of the 8.33% doped system.
 270 The C and B DOS are similar at and above E_F ,
 271 but the C DOS is somewhat lower in the interval

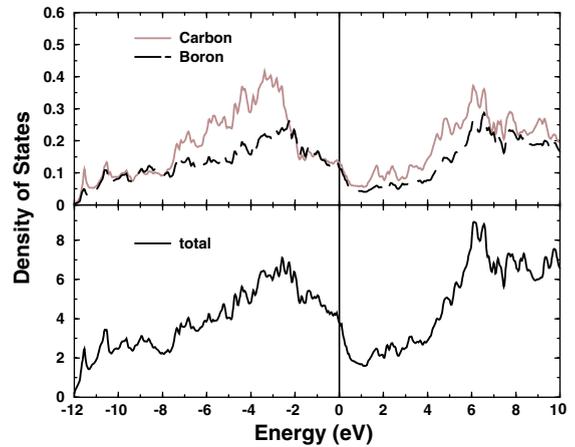


Fig. 4. Density of states, total (bottom) and decomposed into B and C contributions on a per-atom basis, for the ordered $x = 1/12$ model. The primary B-C difference is that the C DOS is lower in the -2 eV to E_F region, but higher in the lower region -8 eV to -2 eV.

272 within 2 eV below E_F . The C DOS is larger than
 273 that of B in the -8 eV to -3 eV region. Based
 274 on charge within spheres of 1.65 Å radius we cal-
 275 culated a charge transfer of about $0.095e^-$ to the
 276 carbon from the three first nearest neighbor boron
 277 atoms, with other B sites showing negligible
 278 change in charge. This charge transfer also reflects
 279 the stronger potential and larger electronegativity
 280 of C, and provides substitutional C with definite
 281 anionic character.

4. 10% Doping

283 We have also performed a virtual crystal calcula-
 284 tion for the 10% doped $x = 0.10$ system $Mg(B_{0.9}C_{0.1})_2$.
 285 The DOS and band structure are shown in Figs. 5
 286 and 6. By aligning the highest peaks in the DOS at
 287 -2 eV and $+6$ eV (they can be aligned simulta-
 288 neously), we establish that the differences are (1)
 289 an increase in the occupied bandwidth (at the
 290 MgB_2 band filling) from 12.4 eV to 12.5 eV
 291 ($\sim 1\%$), and (2) the raising of the Fermi level by
 292 0.3 eV to accommodate the extra electrons. Other-
 293 wise there is very little difference in the two den-
 294 sities of states. This raising of E_F will be compared to
 295 the supercell result in the next section. At this band
 296 filling, the VCA gives the σ -band edge at Γ precisely

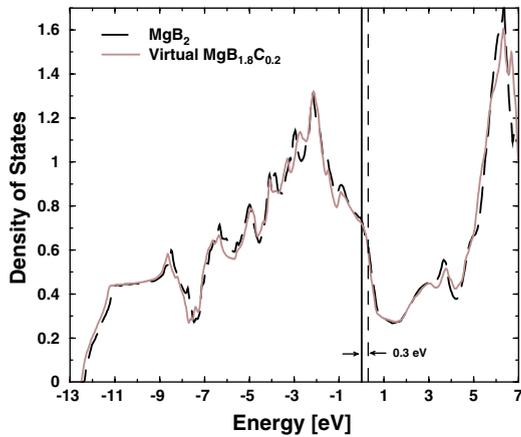


Fig. 5. Density of states of $\text{Mg}(\text{B}_{0.9}\text{C}_{0.1})_2$. The Fermi level moves up by 0.3 eV due to doping.

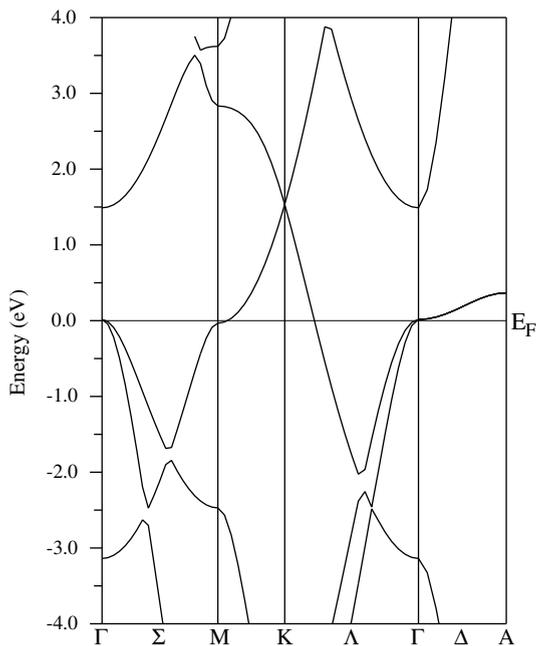


Fig. 6. Band structure of $\text{Mg}(\text{B}_{1-x}\text{C}_x)_2$, $x = 0.10$, in the virtual crystal approximation. The σ -bands at Γ lie precisely at E_F , so the Fermi cylinder radii at Γ vanish, corresponding to a topological transition.

297 at E_F as shown in Fig. 6. At the point A it is still
 298 about 0.3 eV above E_F , meaning that both cylindrical
 299 Fermi surfaces have shrunk to a point at their
 300 “waists.”

5. σ hole concentration

301

It is accepted that superconductivity in MgB_2 arises from hole-doping of the σ bonding hole states due to the intrinsic chemistry [13] of MgB_2 . The observed decrease in electron–phonon coupling strength, and hence T_c upon C addition, makes the change in hole concentration one of the major points of interest. We have calculated the hole concentration of both the 8.33% supercell system and the 10% virtual system from the volume enclosed by the Fermi surface. Since the radii of the cylindrical Fermi surfaces are only slightly different along the $\Gamma \rightarrow M$ and the $\Gamma \rightarrow K$ directions, we calculated the average basal area by considering the value along the $\Gamma \rightarrow M$ direction only. Also, we assumed a sinusoidal dependence of the Fermi surface along the \hat{c} axis, allowing analytic evaluation of the Fermi surface volumes i.e. the hole concentrations. The results are presented in Table 2. In the process of calculating the number of holes, electronic structure reveals that both the σ -band Fermi surfaces are still intact, consistent with the two-band superconductivity with substantial T_c as seen in experiments [8]. Also, we interpolated the number of holes for the 10% doped system (virtual crystal) to calculate the value for the 8.33% doped system. Supercell calculation gave 0.070 holes/cell while the extrapolation gave only 0.057 holes/cell. This indicates that the virtual crystal approximation is not very reliable for substitutional carbon, and that C cannot be thought of “boron + an electron.”

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6. Coherent potential approximation results

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For further comparison and to assess the effects of disorder, we have performed CPA studies of

334
 335

Table 2
 Calculated number of σ holes for the various C concentrations discussed in the text

	No. of holes (holes/cell)
MgB_2	0.11
8.33% Doping $\text{Mg}_6\text{B}_{11}\text{C}$ supercell	0.070
10% Doping $\text{MgB}_{1.8}\text{C}_{0.2}$ virtual crystal	0.0463
8.33% Doping $\text{MgB}_{1.833}\text{C}_{0.167}$ extrapolation	0.057

336 this system. We have carried out CPA calculations
 337 (1) at $x = 0.0001$ to provide a “perfect crystal” ref-
 338 erence for evaluation of the various algorithms in
 339 the CPA code, (2) at $x = 0.0833$ for most direct
 340 comparison to the $x = 1/12$ (ordered) supercell cal-
 341 culations described in the previous sections, and
 342 (3) at $x = 0.10$ and 0.20 as representative of the
 343 system toward the upper range of achievable C
 344 substitution. The spectral function over the whole
 345 energy range of interest, plotted as a “smeared”
 346 band structure, is shown for the $x = 0.20$ case in
 347 Fig. 7. The primary points of interest are the filling
 348 of the σ -band hole states, and the broadening (and
 349 potentially splitting) of bands.

350 The full energy region is presented in Fig. 7 for
 351 $x = 0.20$ (where broadening is more easily seen) to
 352 illustrate the strong wavevector (k) and energy (E)
 353 dependence of the broadening of the spectral func-
 354 tion. The largest disorder occurs in the 2s region in
 355 the lower valence band, where the C 2s state is
 356 noticeably lower in energy leading to increased
 357 smearing. The other region of large disorder is
 358 from +3 eV upward, but some states remain com-
 359 paratively sharp. The flat band at 5 eV along Γ - M
 360 and A - L , with strong π character, is the lowest
 361 conduction band that is strongly affected by the
 362 disorder. The bands around the Fermi level,
 363 whether σ or π , are among those less affected by
 364 the chemical disorder. As expected from the virtual
 365 crystal results for $x = 0.10$ from the previous sec-
 366 tion, the σ -band holes are completely filled at

367 $x = 0.20$, as revealed by the flat band along the
 368 Γ - A line lying entirely below E_F . The band filling
 369 behavior versus x is quantified below.

370 In the valence (occupied) bands, disorder
 371 broadening is large in the lower s-band region
 372 -8 eV to -14 eV below E_F , and somewhat less
 373 so where the (primarily σ) band in the -3 to
 374 -4 eV range gets flat around the zone edge M
 375 and L points. Similar broadening does not occur
 376 in the same band at the (more distant from Γ) zone
 377 edge K and H points, where the band lies at -6 eV
 378 and is much less flat. The top of the σ bonding
 379 bands along Γ - A are comparatively sharp; the
 380 width is quantified below.

381 In the conduction bands disorder broadening
 382 becomes more prevalent. The antibonding σ^* -
 383 bands (flat along Γ - A at +6 eV) are much broader
 384 than their bonding counterparts (below E_F), and
 385 this band along Γ - M and A - L with Mg 3s char-
 386 acter becomes exceedingly diffuse. The π -bands show
 387 strong k -dependence of the broadening, beginning
 388 at 2–3 eV around the M point and becoming wider
 389 in the 6–10 eV range (and above, not shown in the
 390 figure).

391 6.1. Carbon concentration dependence

392 The x -dependence of the broadening can be
 393 seen in Fig. 8 for $x = 0.0001$ (the CPA equivalent
 394 of $x = 0$ MgB₂), for $x = 0.10$, and for $x = 0.20$
 395 (the latter may not be experimentally accessible).
 396 Since in each case E_F is set to zero, band filling ap-
 397 pears as downward shifts of the bands, by roughly
 398 $\delta E_F = -0.4$ eV for $x = 0.10$ and $\delta E_F = -1.0$ eV for
 399 $x = 0.20$. For $x = 0.0833$ (not shown) the σ -bands
 400 lie at the same position (with respect to E_F) as for
 401 the ordered $x = 1/12$ case shown in Fig. 3. Hence
 402 the degree of band filling in the CPA results is
 403 the same as for the ordered $x = 1/12$ case that
 404 was analyzed in the previous section. Although
 405 we will not dwell on it, it can be noticed that the
 406 band shift is not entirely rigid: the two valence
 407 bands at the L point split apart as well as broaden
 408 with increasing C concentration.

409 The other important aspect of the CPA bands
 410 of Fig. 8 is the broadening that increases with car-
 411 bon concentration. Within the resolution of widths
 412 that we are able to extract, the widths of the σ -

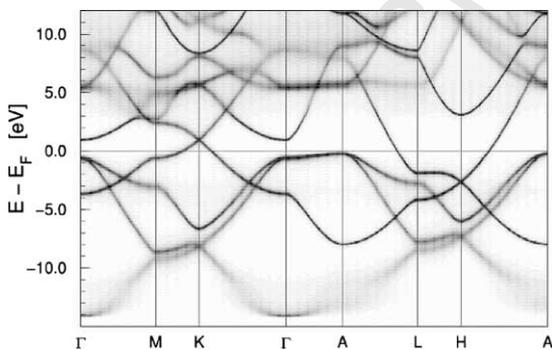


Fig. 7. CPA spectral density in the full valence-conduction band region, for $x = 0.20$, plotted as a broadened band structure. Disorder broadening is largest below -8 eV in the valence bands and above 4 eV in the conduction bands.

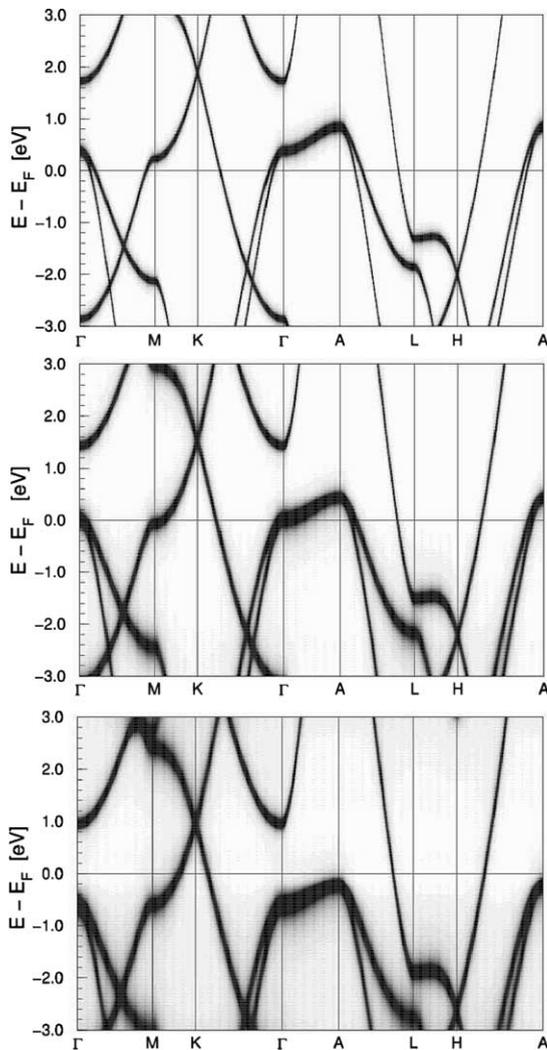


Fig. 8. CPA spectral density for $x=0.0001$ (top panel), $x=0.10$ (middle) and $x=0.20$ (bottom panel). The main features are (i) the “rising” Fermi level (with respect to the σ -band along Γ - A , say), and (ii) the increase in the disorder broadening of the bands. Since the numerical algorithms cannot reproduce the δ -function bands for $x \rightarrow 0$, the $x=0.0001$ case is included as a reference for the algorithmic contribution to the width.

bands can be taken to be proportional to x , but differ considerably between Γ and A (being about 50% wider at Γ). The full width at half maximum of the spectral density of the σ -bands, averaged between Γ and A , is about $\gamma \approx 0.21$ eV (to perhaps 10% accuracy). This width corresponds to a width

in wavevector given by $\gamma = v_F \delta k$, and therefore a mean free path of $\ell_F = 2\pi v_F / \gamma$. The 60 meV splitting of the σ -bands (see Section 3.1) for the $x=1/12$ ordered supercell implies that the 0.21 eV mean width can be interpreted as 0.05 eV from the B/C on-site energy difference, and thus 0.16 eV from disorder itself.

Due to the anisotropy of the Fermi velocity v_F , the mean free path may vary considerably over the Fermi surface at $x=0.10$. At Γ the cylinder radius has shrunk to a point, and the very small z component of v_F (vanishing at Γ) suggests a very small mean free path in the z direction of the order of the layer spacing c for $x=0.10$.

7. Discussion and summary

In the band structure of the 8.33% doped supercell system, the hole σ -band Fermi surfaces along the $\Gamma \rightarrow A$ direction are still present, and this degree of band filling is reproduced by the CPA. Even for $x=0.10$ the virtual crystal picture (which overestimates the rate of band filling by C) leaves σ Fermi surfaces, just beginning to be pinched off. The CPA calculations (Fig. 8) show the σ -band holes begin to disappear rapidly for $x > 0.10$. Qualitatively this filling is consistent with most experimental reports.

The questions, and experimental probes, should now be focused on the fact that, as the σ -band fills, there will be very strong deviation from ‘business as usual’ in the coupled electron–phonon system. The strength of coupling of bond-stretching modes with $Q < 2k_F$ continues to increase, and conventional Migdal–Eliashberg theory ceases to apply. The dynamics of these ultra-strongly coupled modes is unexplored, with their peculiar character being signaled by the divergence of their linewidth (at least within Migdal–Eliashberg theory). The limiting behavior should not revert to the widely studied polaron limit, however, as there remain the background π electrons, which are weakly coupled to vibrations but provide full metallic conductivity and screening to the system. The present study reveals that C substitution for B provides a similarly favorable system to Al substitution for Mg for studying this evolution. Recent reports

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464 indicate that Sc substitution ($\text{Mg}_{1-x}\text{Sc}_x\text{B}_2$) may
465 also provide [32] another such system for study.

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